INTERACTION BETWEEN VIBRATION AND FRICTION

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It is known that an object can be moved with a lower force if there are vibrations. Sometimes we utilise an impact to move the object. If there is friction a model is mathematically highly nonlinear, discontinuous and non-smooth. The problem of interaction between an object and the vibrating base has not been investigated in depth and this research tries to gives some additional information. It is easier to move an object with vibration than without it, therefore an equivalent coefficient of friction is introduced.

The vibration of the base has three components; two in the plane of the base and one perpendicular to it. Each component can change in any way but in this article they are harmonic with the same frequency ω.

$$x_o(t) = a_x \sin(\omega t), \quad y_o(t) = a_y \sin(\omega t + \varphi_y), \qquad z_o(t) = a_z \sin(\omega t + \varphi_z) \qquad (1)$$

The base can be horizontal or inclined. The motion of the object along the base depends on the force P which tries to move the object, the inertial forces I_x, I_y, I_z from the vibration of the base and the friction force.

$I_x(t) = -m\ddot{x}_o = m\omega^2 a_x \sin(\omega t)$ and similar inertial forces for other components of vibration.

The Coulomb model of the friction is taken as

$$\text{if } \dot{x} \neq 0 \quad \text{then } F = -F_o sign(\dot{x}) \quad \text{else} \quad F = abs(P + I_x) sign(P + I_x), \qquad (2)$$

where $F_o = \mu_o N$. The normal force N depends on the normal component of the base vibration $z(t)$ and it is assumed that the object does not lose the contact with the base.

The motion of the object along the horizontal base is governed by

$$m\ddot{x} = P + I_x - F_x \qquad m\ddot{y} = I_y - F_y, \qquad N = mg - I_z > 0$$
(3)

The components of the friction force F_x, F_y depend on the slipping velocity or the direction of the force in the plane Oxy.

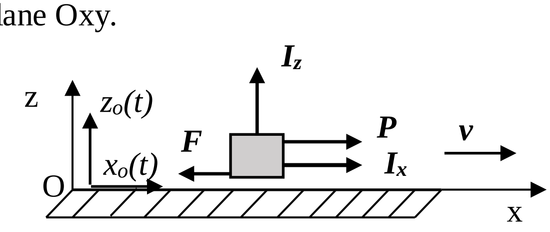

Fig.1.Object on the vibrating plane

For harmonic vibration only in the horizontal direction $x_o(t)$ the minimum force P_{min} to move the object is defined by

$$P_{\min} - F_o + ma_x\omega^2 \geq 0. \qquad \text{where} \qquad F_o = \mu_o mg. \tag{4}$$

It is possible to move the block if $P_{\min} > F_o - ma_x\omega^2$, the inertial force I_x causes an easier motion of the block. If the traditional definition of the coefficient of friction is used $\mu = P_{\min} / N$ then

$$\mu = \frac{\mu_o mg - ma_x\omega^2}{mg} = \mu_o - a_x\omega^2 / g = \mu_o - r \tag{5}$$

where $r = a_x\omega^2 / g$. In this way the defined coefficient of friction has different magnitude than μ_o. It is called the ***equivalent coefficient of friction*** and its magnitude depends on the amplitude and the frequency of vibration of the base as is shown in Fig.2.

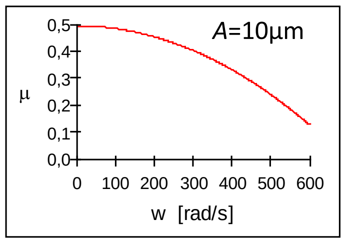

Fig. 2. Equivalent coefficient of friction vs frequency

If the applied force P is from the range $P_{min} < P < \mu_o mg$ then the object moves with a constant average velocity as is shown in Fig.3

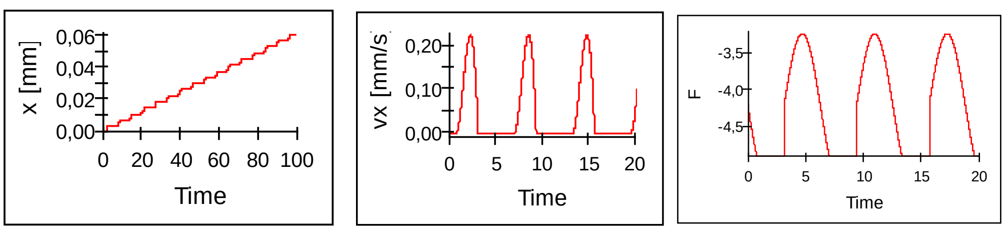

Fig.3. Displacement, velocity and friction force vs. time

When three components of vibration of the base have the same frequency and shift angle then the minimum force is defined by

$$P_{\min} = m\sqrt{(\mu_0(g - a_z\omega^2))^2 - (a_y\omega^2)^2} - ma_x\omega^2 \tag{6}$$

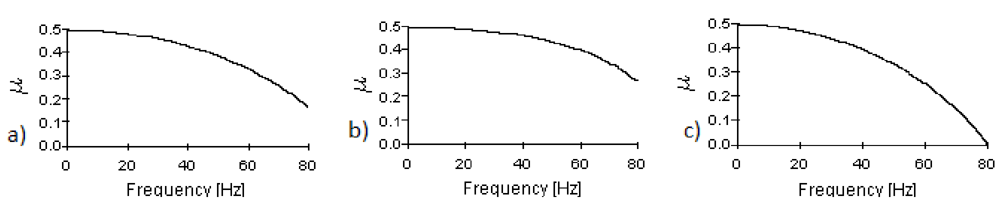

Fig. 3. Coefficient of friction vs. frequency:
a) a_x = 10 µm, a_y = 0, b) a_x = 0, a_y = 10 µm, c) $a_x = a_y$ = 10 µm

When the pushing/pulling force is from $\langle P_{\min}, F_0 \rangle$ range then the block moves with a small constant average velocity – creep motion. By knowing the mechanism of interaction between the two bodies, the phenomena of friction can be better understood. This knowledge can be useful in practice and the resultant friction can be controlled by introducing a specific vibration. On the other hand undesired vibrations can change the magnitude of the coefficient of friction during its measuring.