Karlsruhe Institute of Technology

Universität Karlsruhe (TH)
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Institut für
Technische Mechanik

Friction induced flutter instability
- on modeling and simulation of brake squeal -

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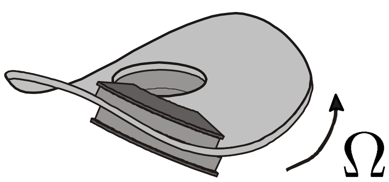
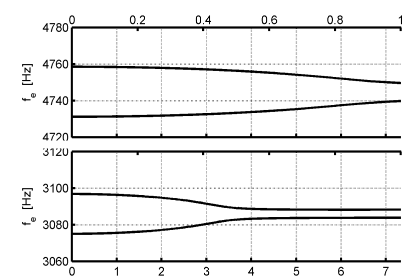
GAMM 2008 / Bremen



- moving continua, contact forces
- stability assessment
- influence of transport motion & damping
- influence of normal contact modelling

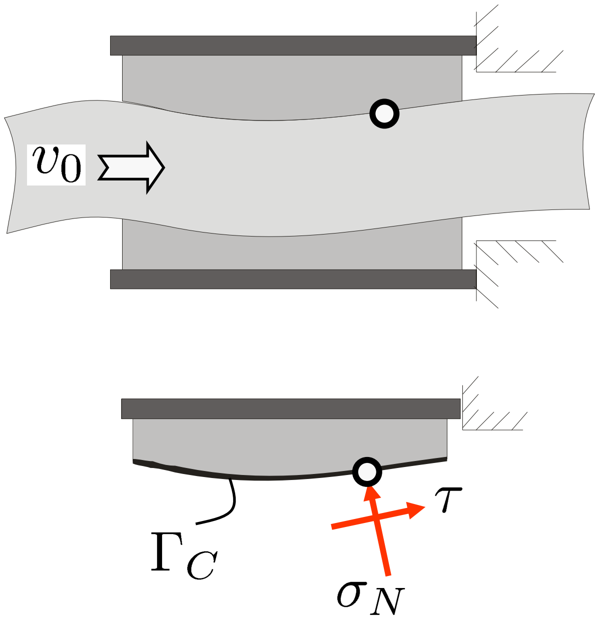


Engineering Problem

Disc brakes (still) squeal !

- system of elastic continua
- moving (some of them)

- contacts (closed)
- sliding friction



Hamilton's Principle

$$\boxed{\delta \int_0^t \{L - \Pi_C\}\,\mathrm{d}t + \int_0^t \{\delta W_{pl} + \delta W_{\Gamma_C}\}\,\mathrm{d}t = 0} \qquad L = T - U$$

normal contact

tangential contact
sliding friction

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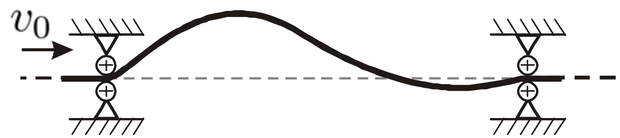
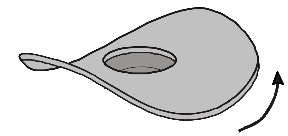
Discretisation

Transport motion



ex.: moving string, fixed boundaries

analogously:
beams, plates, …



Discretization

- moving continua (with spatially fixed boundaries)
- description in an Eulerian frame

ansatz: $\vec{p}_i(\mathbf{x},t) \approx \mathbf{\Phi}_i(\mathbf{x})\mathbf{q}_i(t)$, $\dot{\vec{p}}_i(\mathbf{x},t) \approx \mathbf{\Phi}_i(\mathbf{x})\dot{\mathbf{q}}_i(t)$ . . .

H's P: $$\int_0^t \{\delta L + \delta W_{pl} + \delta W_{\Gamma_C} - \delta\Pi_C\}\,\mathrm{d}t = 0$$

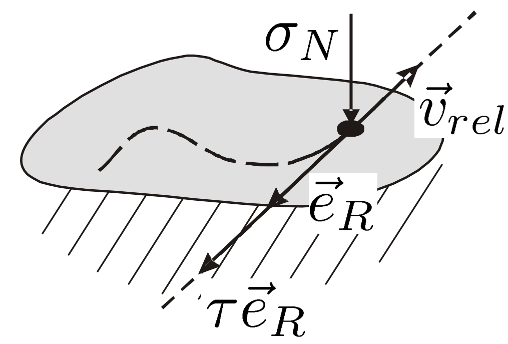
discretization

$$\delta\mathbf{q}^\top\Big\{\mathbf{M}\ddot{\mathbf{q}} + [\mathbf{G} + \mathbf{D}]\,\dot{\mathbf{q}} + \mathbf{K}\mathbf{q}\Big\} = \delta\mathbf{q}^\top\Big\{\mathbf{f}(\mathbf{q},\dot{\mathbf{q}},t)\Big\} - \delta\bar{W}_{\Gamma_C} - \delta\bar{\Pi}_C$$

transport motion | sliding friction | normal contact

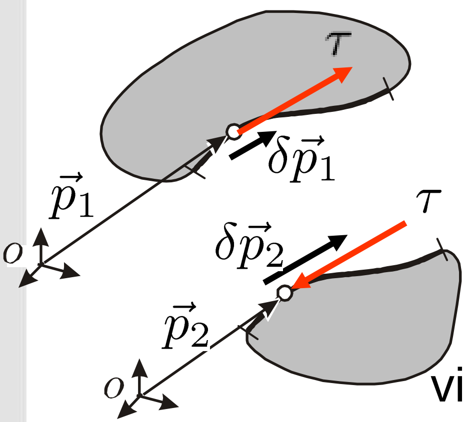
Sliding friction

$$\int_0^t \{\delta L + \delta W_{pl} - \delta\Pi_C + \boxed{\delta W_{\Gamma_C}}\}\,\mathrm{d}t = 0$$



$$\boxed{\delta W_{\Gamma_C} = \int_{\Gamma_C} \underbrace{\tau\,\vec{e}_R}\cdot\delta g_T\,\mathrm{d}A}$$

$$\vec{e}_R = -\frac{\vec{v}_{rel}}{|\vec{v}_{rel}|}$$



e.g. Coulomb

orientation $\vec{e}_R = \vec{e}_R(\vec{v}_{rel})$

value $\tau = \mu\sigma_N \longrightarrow \sigma_N$ **from contact-law**

virt. displacement $\delta g_T = \delta\vec{p}_i - \delta\vec{p}_j \longrightarrow$ **contact kinematics**

$$\boxed{\delta W_{\Gamma_C} = \delta W_{\Gamma_C}(\mu, \vec{p}_i, \vec{v}_i, \sigma_N)}$$

depending on

- positions- and velocities
- coefficient of friction
- contact law

Normal contact / contact discretization

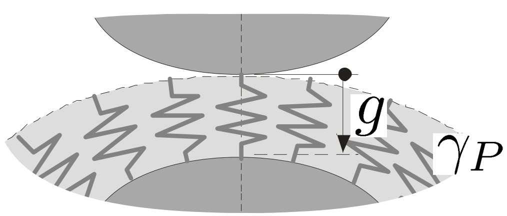
$$\int_0^t \{\delta L + \delta W_{pl} + \boxed{\delta \Pi_C} + \delta W_{\Gamma_C}\}\,\mathrm{d}t = 0 \qquad \delta W_{\Gamma_C} = \delta W_{\Gamma_C}(\mu, \vec{p}_i, \vec{v}_i, \sigma_N)$$

Lagrange-multipliers $\Pi_C = \int_{\Gamma_C} \lambda_N g \mathrm{d}a \Rightarrow \delta\Pi_C = \int_{\Gamma_C} \delta\lambda_N g \mathrm{d}a + \int_{\Gamma_C} \lambda_N \delta g \mathrm{d}a$

- ***usually***: discretization $\lambda_N \approx \bar{\lambda}_N \quad g_N \approx \bar{g}_N \rightsquigarrow$ DAE
- ***in simple cases*** by ansatz functions $\bar{g} \equiv 0$, $\delta\bar{g} \equiv 0 \Rightarrow \boxed{\delta\bar{\Pi}_C^{(L)} = 0} \quad \boxed{\delta\bar{W}_{\Gamma_C}^{(L)} = \delta\bar{W}_{\Gamma_C}^{(L)}(\mu, \mathbf{q}, \dot{\mathbf{q}})}$

$$\sigma_N = \sigma_N(\vec{\varepsilon}) \qquad \vec{\varepsilon} = \mathrm{grad}\vec{p}$$

Regularization (Penalty) $\Pi_C = \frac{1}{2}\int_{\Gamma_C} \gamma_P g^2 \mathrm{d}a \Rightarrow \delta\bar{\Pi}_C = \delta\mathbf{q}^\top \left\{ \int_{\Gamma_C} \gamma_p \mathbf{\Theta}^\top \mathbf{\Theta} \mathrm{d}a \right\} \mathbf{q}$



penalty parameter

≅ *„contact stiffness“*

$$\sigma_N = \gamma_P g_N \approx \gamma_P \mathbf{\Theta}\mathbf{q}$$

$$\boxed{\delta\bar{\Pi}_C^{(P)} = \delta\mathbf{q}^\top \gamma_P \mathbf{K}_P\, \mathbf{q}} \quad \mathbf{K}_P = \mathbf{K}_P^\top$$

$$\boxed{\delta\bar{W}_{\Gamma_C}^{(P)} = \delta\bar{W}_{\Gamma_C}^{(P)}(\mu, \mathbf{q}, \dot{\mathbf{q}}, \gamma_P)}$$

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Linearized equations of motion

$$\delta\mathbf{q}^\top\Big\{\mathbf{M}\ddot{\mathbf{q}} + [\mathbf{G}+\mathbf{D}]\,\dot{\mathbf{q}} + \mathbf{K}\mathbf{q}\Big\} = \delta\mathbf{q}^\top\Big\{\mathbf{f}(\mathbf{q},\dot{\mathbf{q}},t)\Big\} - \boxed{\delta\bar{W}_{\Gamma_C}} - \boxed{\delta\bar{\Pi}_C}$$

constraints by ansatz functions

$$\underline{\delta\bar{\Pi}_C^{(L)} = 0} \quad \delta\bar{W}_{\Gamma_C}^{(L)} = \delta\bar{W}_{\Gamma_C}^{(L)}(\mu,\mathbf{q},\dot{\mathbf{q}})$$

$$\mathbf{M}\ddot{\mathbf{q}} + [\mathbf{G}+\mathbf{D}]\,\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \mathbf{f}(\mathbf{q},\dot{\mathbf{q}},t) + \underline{\mathbf{f}_R(\mu,\mathbf{q},\dot{\mathbf{q}})}$$

$$\boxed{\mathbf{M}\ddot{\mathbf{q}} + [\mathbf{G}+\mathbf{D}+\boxed{\mu\mathbf{P}_R}]\,\dot{\mathbf{q}} + [\mathbf{K}+\boxed{\mu\mathbf{Q}_R}]\,\mathbf{q} = \mathbf{f}(\mathbf{q},\dot{\mathbf{q}},t)}$$

Regularization (Penalty)

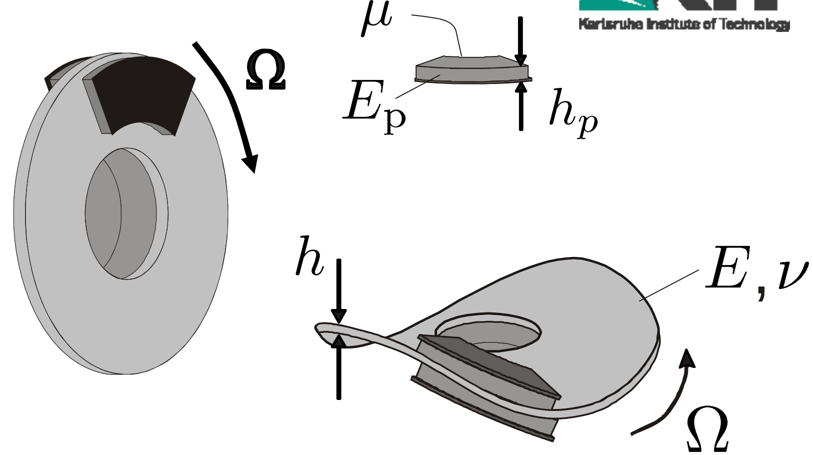
$$\delta\bar{\Pi}_C^{(P)} = \delta\mathbf{q}^\top\gamma_P\mathbf{K}_P\,\mathbf{q} \quad \delta\bar{W}_{\Gamma_C}^{(P)} = \delta\bar{W}_{\Gamma_C}^{(P)}(\mu,\mathbf{q},\dot{\mathbf{q}},\gamma_P)$$

$$\mathbf{M}\ddot{\mathbf{q}} + [\mathbf{G}+\mathbf{D}]\,\dot{\mathbf{q}} + [\mathbf{K}+\underline{\gamma_P\mathbf{K}_P}]\,\mathbf{q} = \mathbf{f}(\mathbf{q},\dot{\mathbf{q}},t) + \underline{\mathbf{f}_R(\mu,\mathbf{q},\dot{\mathbf{q}},\gamma_P)}$$

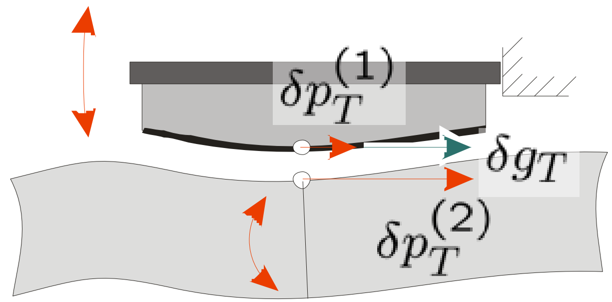
$$\boxed{\mathbf{M}\ddot{\mathbf{q}} + [\mathbf{G}+\mathbf{D}+\boxed{\mu\underline{\gamma_P}\mathbf{P}_R}]\,\dot{\mathbf{q}} + [\mathbf{K}+\boxed{\underline{\gamma_P}\mathbf{K}_P}+\boxed{\mu\underline{\gamma_P}\mathbf{Q}_R}]\,\mathbf{q} = \mathbf{f}(\mathbf{q},\dot{\mathbf{q}},t)}$$

Model of a disc brake

- rotating Kirchhoff - plate
- compressible pads
- normal contact kinematically
- detailled contact kinematics $\delta g_T \neq 0$



$$\delta W_{\Gamma_C} = \int_{\Gamma_C} \tau \, \vec{e}_R \cdot \delta g_T \, \mathrm{d}A$$



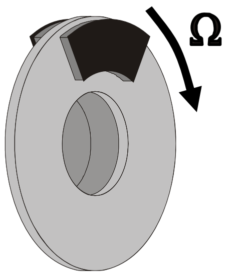
omitting negligible parts ($\Omega^2 \ll 1$ ...)

$$\boxed{\mathbf{M}\ddot{\mathbf{q}} + \left(\Omega \mathbf{G} + d_p \mathbf{D} + p_1 \frac{s_0 h}{\Omega} \mathbf{D}_R\right) \dot{\mathbf{q}} + (\mathbf{K} + p_1 \mathbf{N})\, \mathbf{q} = \mathbf{0}}$$

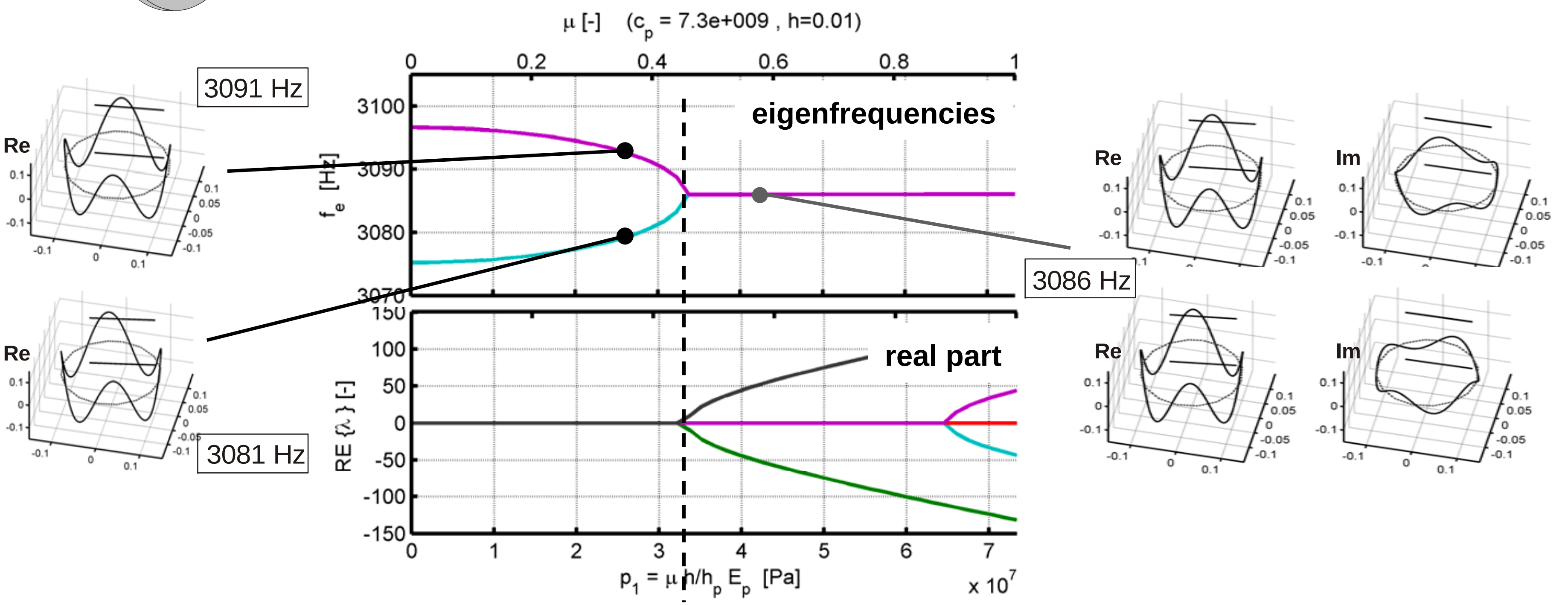
$p_1 = \mu c_p h$
load parameter

matrices: $\mathbf{M}, \mathbf{D}, \mathbf{D}_R, \mathbf{K}$ - symmetric $\mathbf{G}, \mathbf{N}$ - skew-symmetric

brake model (simplified)



$$\mathbf{M}\ddot{\mathbf{q}} + (\mathbf{K} + p_1\mathbf{N})\,\mathbf{q} = \mathbf{0}$$



stable

instable

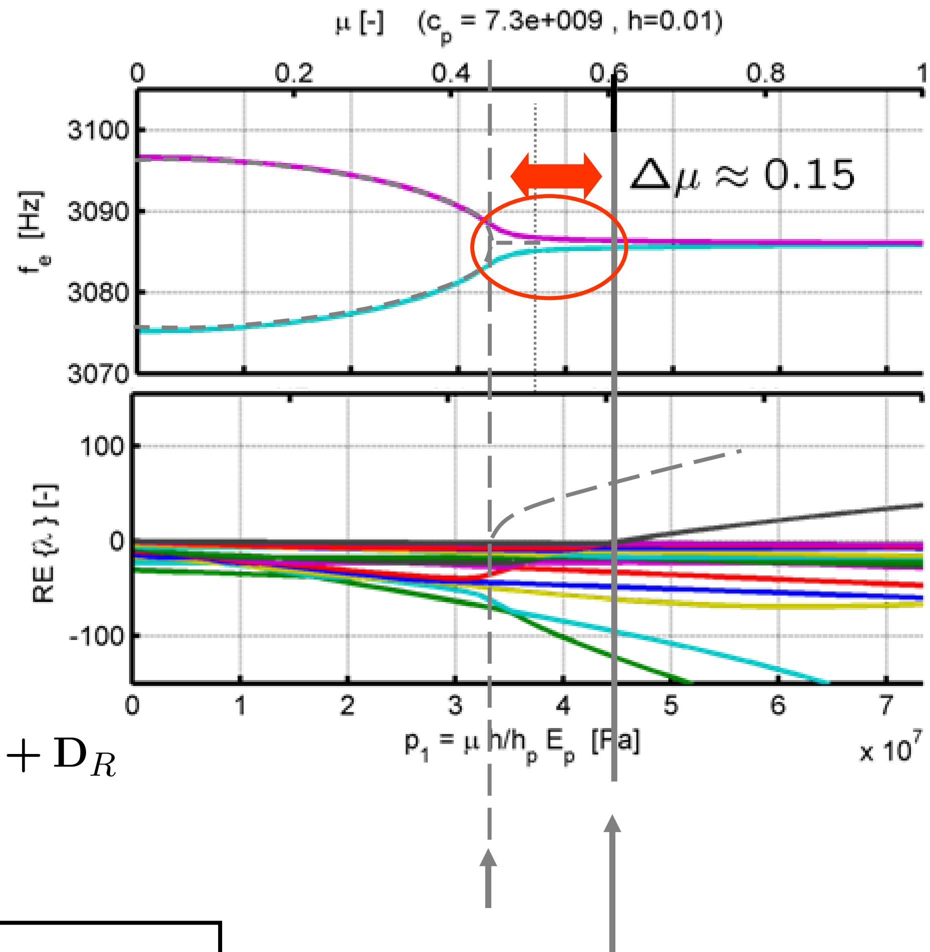
p_{crit}

- real eigenvectors
- different eigenfrequencies

- complex eigenvectors
- on eigenfrequencies

Brake model (complete)

- Thomson-Tait-Chetayev does not apply for circulatory systems
- specific effects in circulatory systems with damping / gyroscopic contributions *(„Ziegler's Paradox")*



stability assessment must account for
- transport motion $\mathbf{G}$
- damping + consistent fritcion linearization $\mathbf{D} + \mathbf{D}_R$

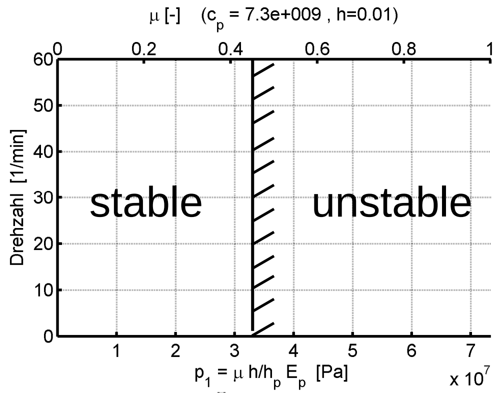
stability border

$$\boxed{\mathbf{M}\ddot{\mathbf{q}}+\left(\Omega\mathbf{G}+d\mathbf{D}+p_1\frac{s_0 h}{\Omega}\mathbf{D}_R\right)\dot{\mathbf{q}}+(\mathbf{K}+p_1\mathbf{N})\,\mathbf{q}=\mathbf{0}}$$

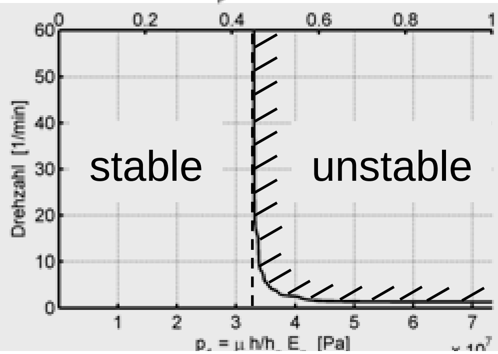
Modelling and stability assessment

motion of disc | friction (vel.) | friction (pos.)

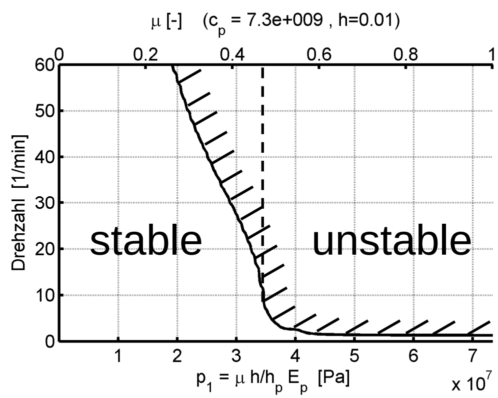
$$\mathbf{M}\ddot{\mathbf{q}}+\left(d\mathbf{D}\right)\dot{\mathbf{q}}+(\mathbf{K}+p_1\mathbf{N})\,\mathbf{q}=\mathbf{0}$$



$$\mathbf{M}\ddot{\mathbf{q}}+\left(d\mathbf{D}+p_1\frac{s_0h}{\Omega}\mathbf{D}_R\right)\dot{\mathbf{q}}+(\mathbf{K}+p_1\mathbf{N})\,\mathbf{q}=\mathbf{0}$$



$$\mathbf{M}\ddot{\mathbf{q}}+\left(\Omega\mathbf{G}+d\mathbf{D}+p_1\frac{s_0h}{\Omega}\mathbf{D}_R\right)\dot{\mathbf{q}}+(\mathbf{K}+p_1\mathbf{N})\,\mathbf{q}=\mathbf{0}$$

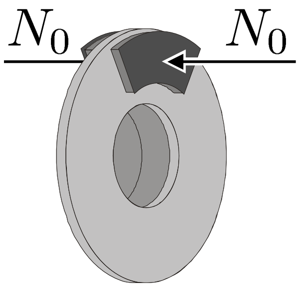


transport motion + complete friction linearization

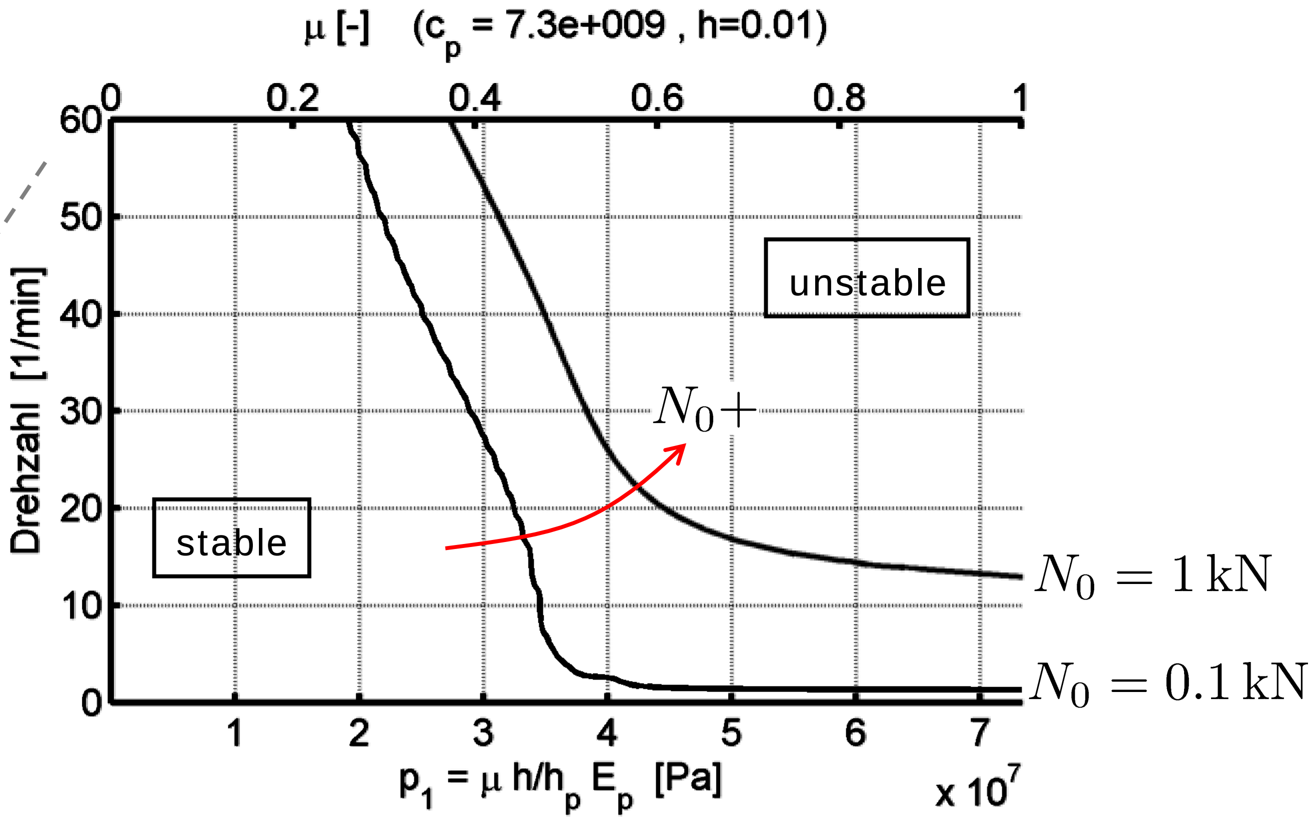
must be considered

Brake model

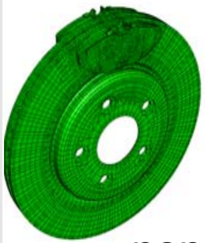
$$\mathbf{M}\ddot{\mathbf{q}}+\left(\Omega\mathbf{G}+d_p\mathbf{D}+p_1\frac{s_0 h}{\Omega}\mathbf{D}_R\right)\dot{\mathbf{q}}+(\mathbf{K}+p_1\mathbf{N})\,\mathbf{q}=\mathbf{0}$$



$60\ 1/\mathrm{min}\ \hat{\approx} 8\ \mathrm{km/h}$



Stability assessment in practice



FE discretization

$$\mathrm{M}\ddot{\mathrm{q}} + [\mathrm{G} + \mathrm{D}]\,\dot{\mathrm{q}} + [\mathrm{K} + \mathrm{N}]\,\mathrm{q} = 0$$

- nonlinear deformations, linearization √
- spatial friction, consistent friction linearization (position and velocity level) √

User:

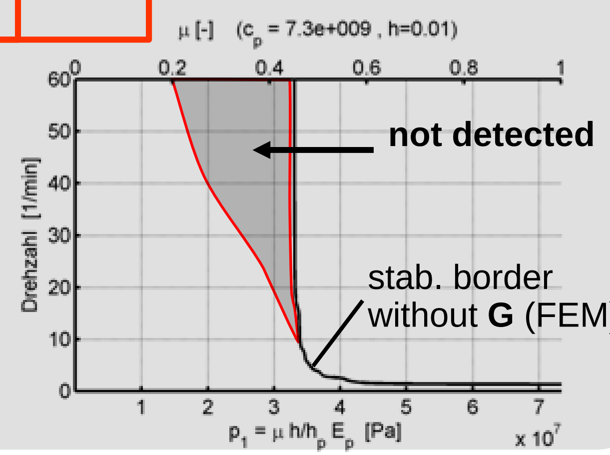
- damping and transport motion often neglected (Thomson-Tait does not apply !)

Software:

often only solves eigenvalue problems

$$\left[\mathrm{M}\lambda^2 + \underline{\mathrm{D}\lambda} + (\mathrm{K} + \mathrm{N})\right]\mathrm{r} = 0$$

$\rightarrow$ $\mathbf{G}$ not considered !

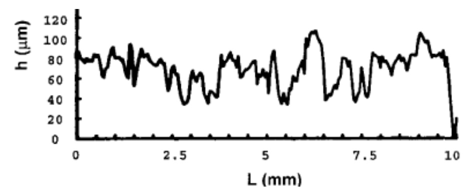


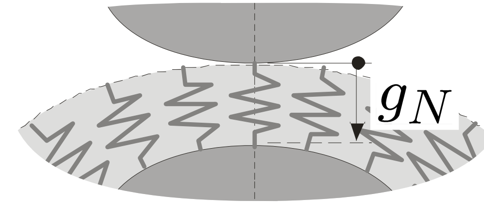
Modeling contacts in practice

often: **regularization** (penalty) to enforce contact

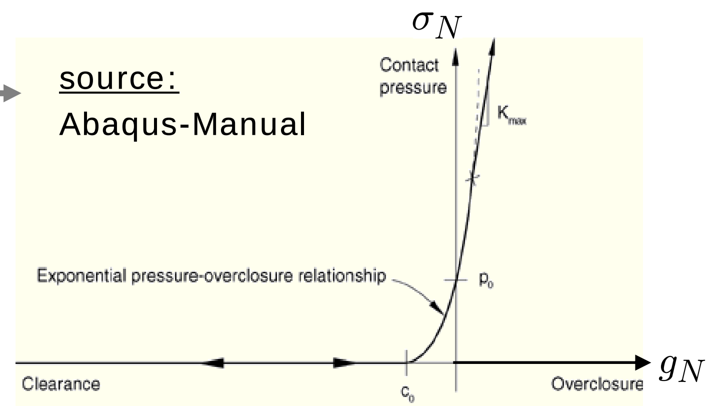
physical interpretation:

stiffness of the contact layer





- Oden-Martins – model $\sigma_N = c_0 (g_N)^m$
- exponential model $\sigma_N = p_0 \mathrm{e}^{\lambda g_N}$
- etc.



linearisation

(linear)
contact stiffness γ_P

$$\boxed{\mathbf{M}\ddot{\mathbf{q}} + \Big([\mathbf{K} + \gamma_P \mathbf{K}_P] + \mu \gamma_P \mathbf{N} \Big) \mathbf{q} = \mathbf{0}}$$

$\boxed{\gamma_P \rightarrow \infty}$

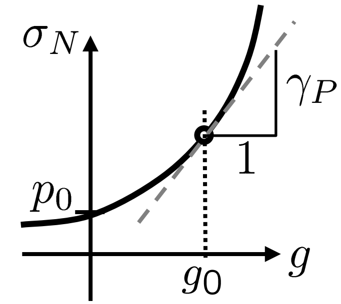
„hard contact“

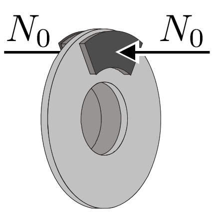
limit case „hard contact“ not sensible !

Stability border

„soft contact“ (exponential) $\boxed{\sigma_N = p_0 \mathrm{e}^{\lambda g_N}}$

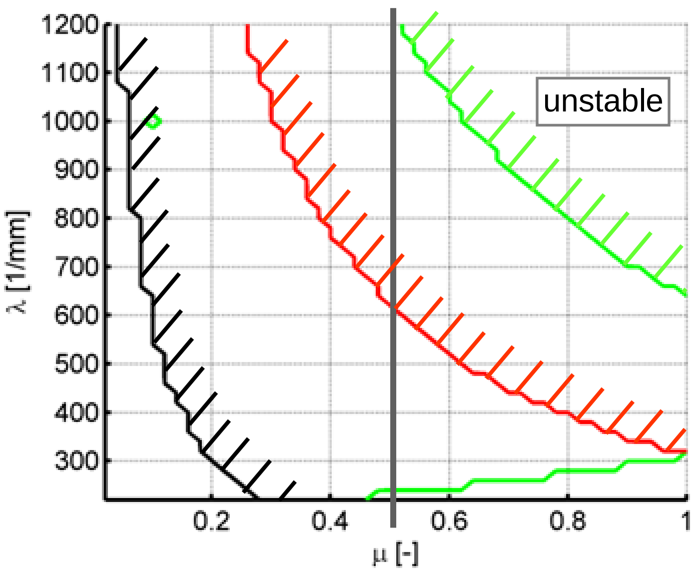
$$\mathbf{M}\ddot{\mathbf{q}} + \left([\mathbf{K} + \gamma_P \mathbf{K}_P] + \mu\gamma_P \mathbf{N}_P\right)\mathbf{q} = \mathbf{0} \qquad \gamma_P = \frac{N_0}{A_p}\lambda$$

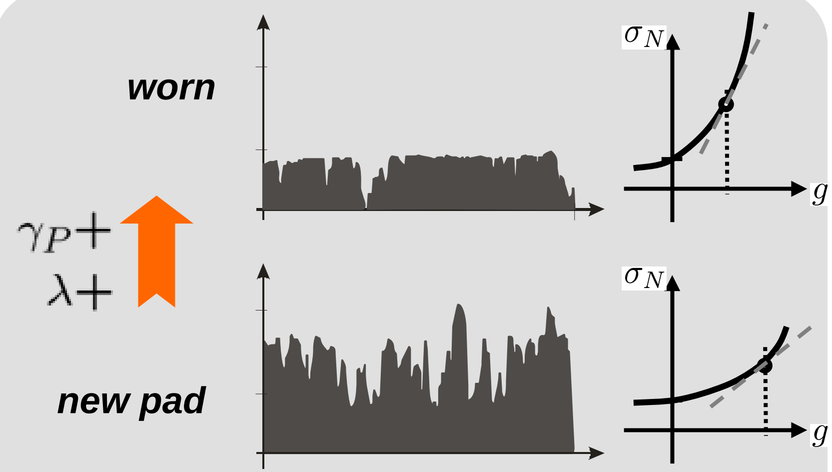




$N_0 = 5 \cdot 10^2$ N
$N_0 = 1 \cdot 10^3$ N
$N_0 = 5 \cdot 10^3$ N

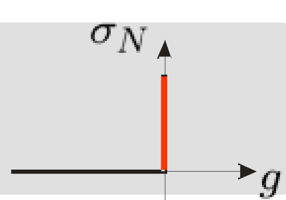
$\boxed{p_0 = \text{const}}$





from: Sherif: *Investigation on effect of surface topography…on squeal generation*, Wear, 2004

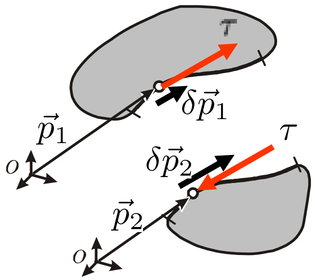
„hard contact“ $\mathbf{M}\ddot{\mathbf{q}} + \left(\mathbf{K} + \mu c_p h \mathbf{N}\right)\mathbf{q} = 0$



Conclusion

moving continua with frictional contacts

- contact modeling
- contribution due to friction
- generic form of linearized equations



application: model of a disc brake

- stability assessment
- influence of transport motion
- assessment in practice
- relevance of a constitutive model for the normal contact