Karlsruhe Institute of Technology

Universität Karlsruhe (TH)
Research University • founded 1825

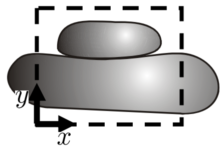
Institut für
Technische Mechanik

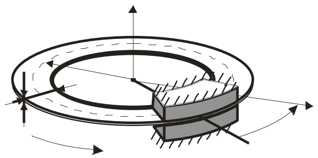
80th annual meeting of GAMM, Gdańsk 2009

On the influence of contact mechanics on friction induced flutter instability

Hartmut Hetzler

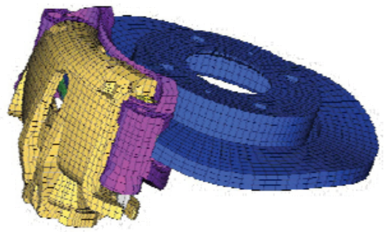
- general perturbation equations for systems of elastic bodies
- linearized contact contributions
- example: rotating Timoshenko annulus
- influence of contact properties



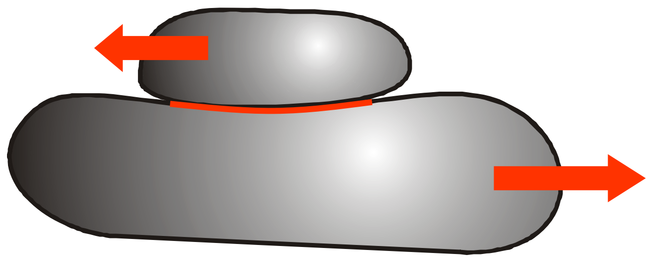


Engineering Problems

Photos: www.technik-handel.com, Bosch

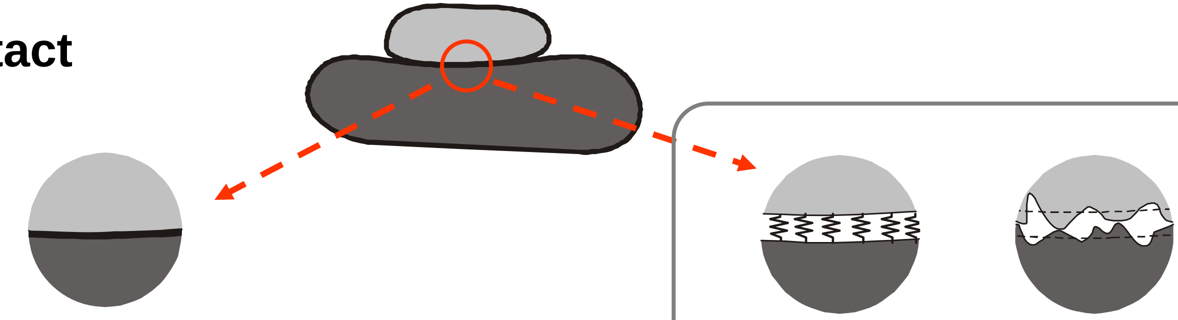


- systems of elastic continua
- relative motion
- sliding friction contacts



System Description

Normal contact



- Lagrange Multipliers:
  „ideal bodies“ → kinematic constraint

- Penalty formulation:
  „contact layer“ → contact stiffness

Hamilton‘s Principle (for open systems)

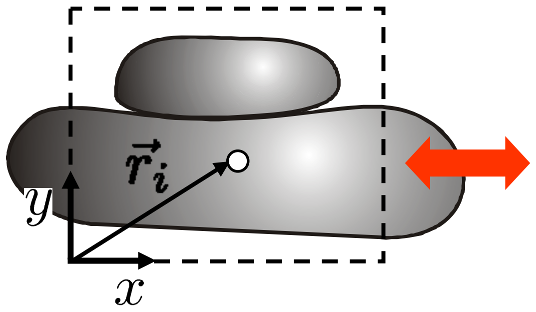
$$\boxed{\int_0^t \{\delta L + \delta W_{np}^* - \delta \Pi_C + \delta W_{\Gamma_C}\}\,\mathrm{d}t = 0} \qquad L = T - U$$

normal contact ($\delta \Pi_C$)

tangential contact
sliding friction (δW_{Γ_C})

Linearized system

$$0 = \int_0^t \left\{ \delta L + \delta W_{np}^{*} - \delta \Pi_C + \delta W_{\Gamma_C} \right\} \mathrm{d}t$$



$\mathbf{x} = (x, y)^\top$

- Eulerian description $\vec{r}_i = \vec{r}_i(\mathbf{x})$
- transport motion relates
  Eulerian to Lagrangian coordinates by $\mathbf{x} = \mathbf{x}\left(\mathbf{X}, t\right)$
- superposition of transport motion and small vibrations about steady state solution
  → Linearization $\vec{r}_i = \vec{r}_{i,0} + \vec{w}_i$

$$0 = \sum_{i=1}^{N} \int_{\Omega_i} \delta \vec{w}_i \cdot \left(\mathcal{M}_i[\ddot{\vec{w}}_i] + \mathcal{P}_i[\dot{\vec{w}}_i] + \mathcal{Q}_i[\vec{w}_i] \right) \mathrm{d}v + \underbrace{\Delta\left\{\delta\Pi_C\right\} - \Delta\left\{\delta W_{\Gamma_C}\right\} - \Delta\left\{\delta W_{np}\right\}}_{\text{linearized contact contributions}}$$

linearized contact contributions

Contact contributions

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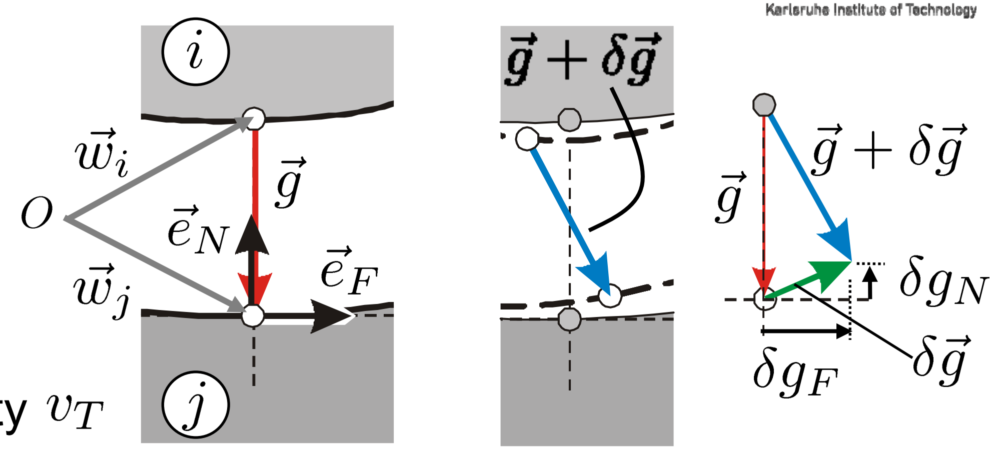
Linearization

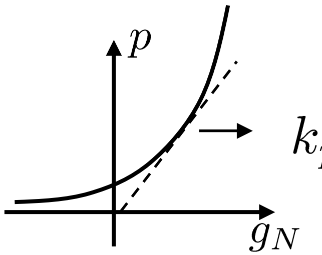
- small motions
- no damping in contact layer
- dry friction, $\mu = \text{const}$
- motion along x-axis, transport velocity v_T



$\vec{e}_F = \vec{v}_{rel}/|\vec{v}_{rel}|$

$$\Delta\left\{\delta\Pi_C\right\} - \Delta\left\{\delta W_{\Gamma_C}\right\}$$

$$= \sum_{(ij)}^{n_C} \int_{\Gamma_C^{(ij)}} \delta\vec{g}\cdot\left(\underbrace{k_p\left(\vec{e}_N\otimes\vec{e}_N\right)\vec{g}}_{\text{normal contact}} + \underbrace{\frac{\mu p_0}{v_T}\dot{\vec{g}} + \mu p_0 \frac{\partial}{\partial x}\vec{g} + \mu k_p\left(\vec{e}_F\otimes\vec{e}_N\right)\vec{g}}_{\text{sliding friction}}\right) \mathrm{d}a$$



$$k_p = \left.\frac{\partial p}{\partial g_N}\right|_0$$

$$\boldsymbol{\vec{e}_N^{(ij)}} = \boldsymbol{\vec{e}_{Nj}}\,,\ \vec{g}^{(ij)} = \vec{w}_j - \vec{w}_i\ ,\ g_N^{(ij)} = \vec{e}_{Nj}\cdot\vec{g}^{(ij)}$$

Influence of contacts

normal contact | **sliding friction**

orientation of friction vector | change of contact pressure

$$= \sum_{(ij)}^{n_C} \int_{\Gamma_C^{(ij)}} \delta\vec{g} \cdot \left(\underbrace{k_p \left(\vec{e}_N \otimes \vec{e}_N\right) \vec{g}}_{\delta g_N (k_p g_N)} + \frac{\mu p_0}{v_T} \dot{\vec{g}} + \mu p_0 \frac{\partial}{\partial x} \vec{g} + \underbrace{\mu k_p \left(\vec{e}_F \otimes \vec{e}_N\right) \vec{g}}_{\delta g_F \mu (k_p g_N)} \right) \mathrm{d}a$$

normal contact	orientation of friction vector		change of contact pressure
stiffness	damping	stiffness	stiffness
sym., pos.def.	sym., pos.def.	non.sym.	non.sym.

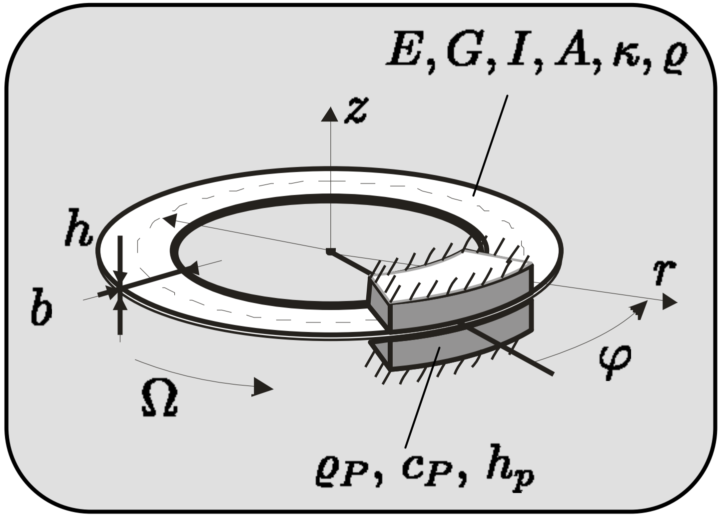
→ **general form of linearized contact contributions**
→ **no discretization so far**

Contact Parameters

physical $\Gamma_C\,,\ k_p\,,\ p_0(x)\,,\ \mu\,,\ v_T$

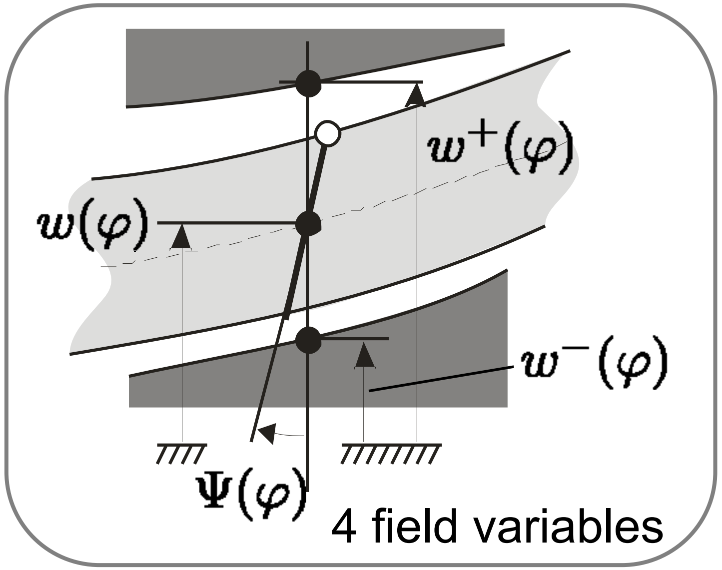
modeling kinematics $\delta\vec{g} \longrightarrow \delta\vec{g}\cdot\vec{g}_F \overset{!}{\neq} 0$

Example: rotating Timoshenko ring



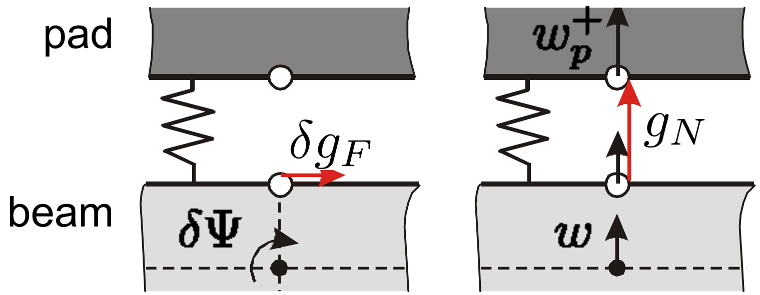
- rotating circular Timoshenko beam
- friction pads as Winkler bedding
- Eulerian description
- simple model for brake squeal

data: $\kappa = 5/6,\ R = 0.12\text{m},\ h = 0.01\text{m},\ b = 0.1\text{m},\ \varphi_0 = \pi/8,$
$c_P = 8 \cdot 10^8 \text{Pa/m},\ h_p = 0.02\text{m},\ k_p = 5 \cdot 10^{10}\ \text{Pa/m},$
$E = 2.1 \cdot 10^{11}\text{Pa},\ \nu = 0.33,\ \varrho = 7800\text{kg/m}^3$



4 field variables

$$= \sum_{(ij)}^{n_C} \int_{\Gamma_C^{(ij)}} \delta \vec{g} \cdot (\ldots + \underbrace{\mu k_p \left(\vec{e}_F \otimes \vec{e}_N\right) \vec{g}}_{\delta g_F \mu \left(k_p g_N\right)})\ \mathrm{da}$$



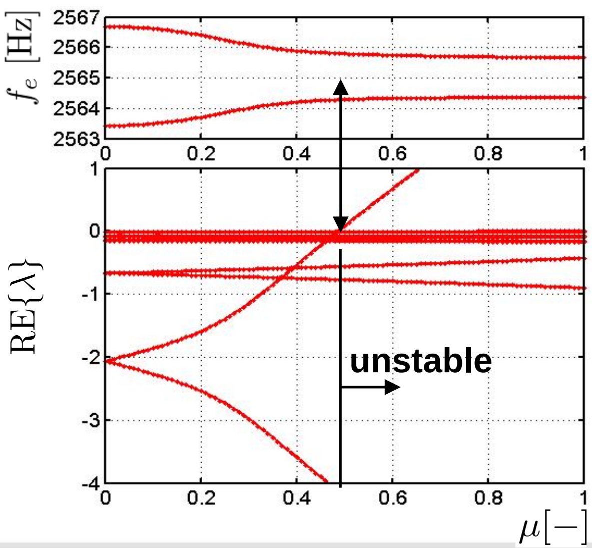
Stability of steady-state

$$\Delta\{\delta\Pi_C\}-\Delta\{\delta W_{\Gamma_C}\}=\sum_{(ij)}^{n_C}\int_{\Gamma_C^{(ij)}}\delta\vec{g}\cdot\left(k_p\left(\vec{e}_N\otimes\vec{e}_N\right)\vec{g}+\frac{\mu p_0}{v_T}\dot{\vec{g}}+\mu p_0\cancelto{\approx 0}{\frac{\partial}{\partial x}}\vec{g}+\mu k_p\left(\vec{e}_F\otimes\vec{e}_N\right)\vec{g}\right)\mathrm{d}a$$

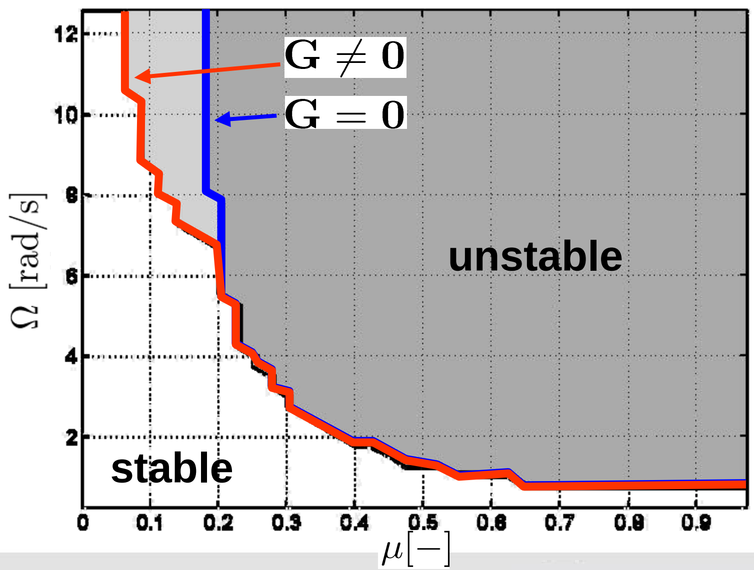
discretization

$$\boxed{\mathbf{M}\ddot{\mathbf{q}}+\left(\Omega\mathbf{G}+\mathbf{D}_S+\frac{\mu p_0}{\Omega}\mathbf{D}_F\right)\dot{\mathbf{q}}+\left(\mathbf{K}_S+k_p\mathbf{K}_N+\mu k_p h\mathbf{Q}_F\right)\mathbf{q}=\mathbf{0}}$$

Flutter-type instability



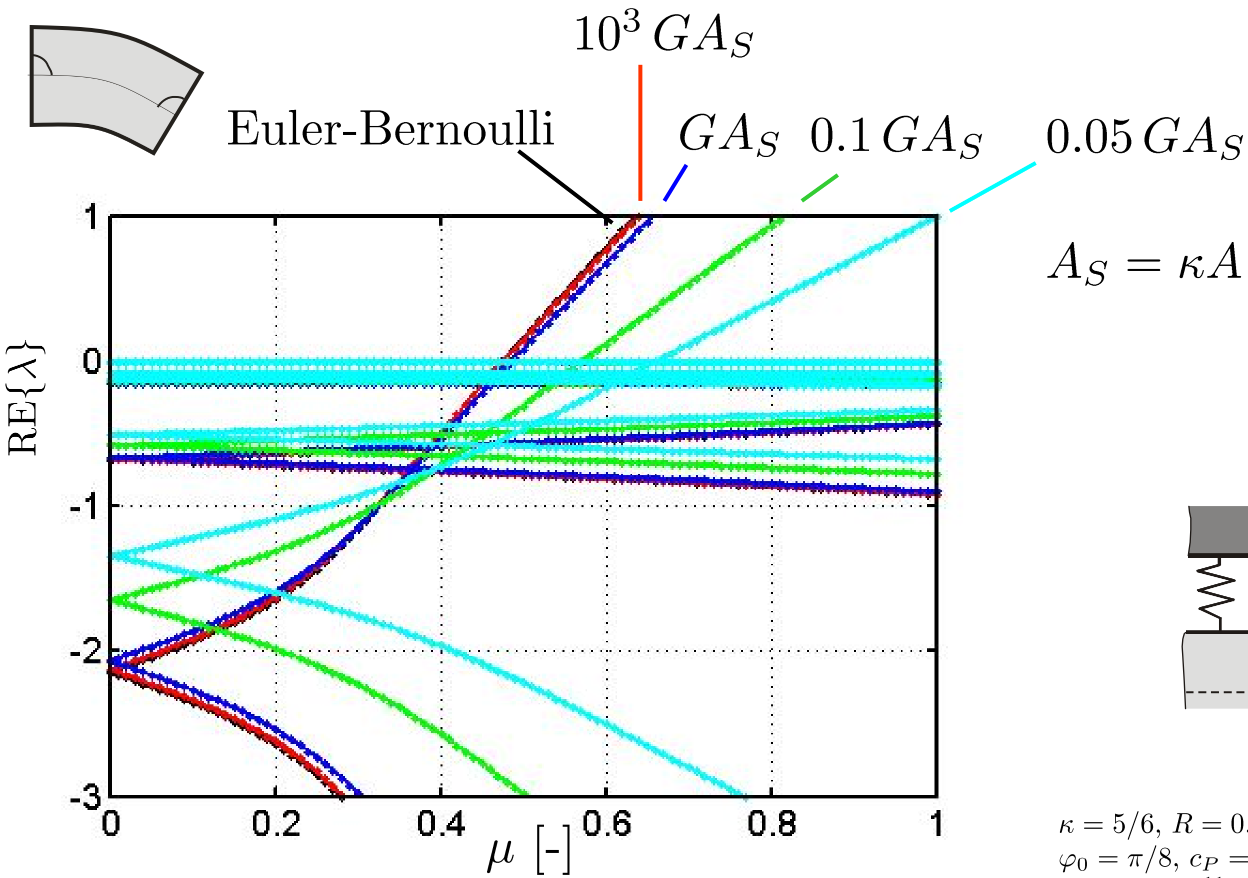
Gyroscopic influence

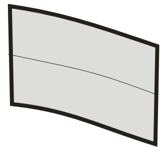


Influence of shear stiffness

$GA_S \to \infty$: Euler-Bernoulli
$w' = -\Psi$

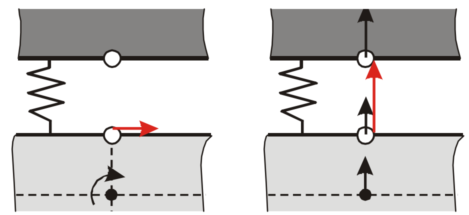
$GA_S \to 0$: decoupling of
w' and Ψ





$A_S = \kappa A$

$$\delta g_F \mu \left(k_p g_N\right)$$

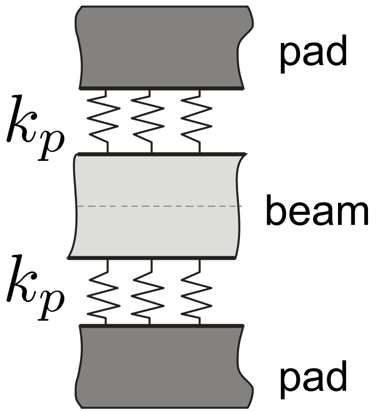


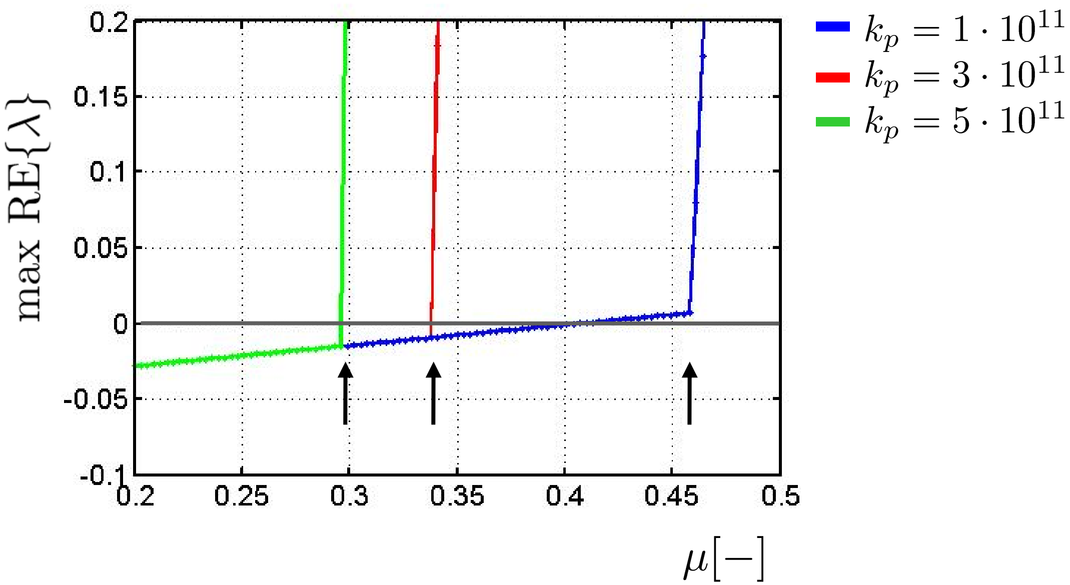
$\kappa = 5/6$, $R = 0.12\text{m}$, $h = 0.01\text{m}$, $b = 0.1\text{m}$,
$\varphi_0 = \pi/8$, $c_P = 8 \cdot 10^8\text{Pa/m}$, $k_p = 5 \cdot 10^{10}$ Pa/m
$E = 2.1 \cdot 10^{11}\text{Pa}$, $\nu = 0.33$

Influence of contact stiffness k_p

$$\mathbf{M}\ddot{\mathbf{q}} + \left(\Omega\mathbf{G} + \mathbf{D}_S + \frac{\mu p_0}{\Omega}\mathbf{D}_F\right)\dot{\mathbf{q}} + \left(\mathbf{K}_S + k_p\mathbf{K}_N + \mu k_p h \mathbf{Q}_F\right)\mathbf{q} = \mathbf{0}$$

$$k_p \neq f(x)$$



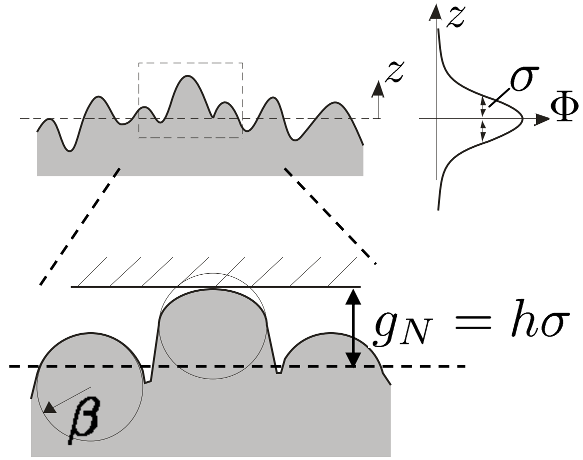


→ contact stiffness has significant influence

Constitutive contact law

Greenwood-Williamson

- height distribution Φ
- asperities as spherical caps, radius β
- deformation: Hertz-theory
- here: Gaussian distribution, std. deviation σ

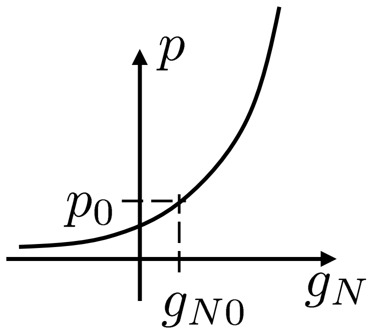


$$p(h) = \frac{4}{3}\eta E^* \sqrt{\beta\sigma^3} \int_h^\infty (s-h)^{3/2}\Phi(s)\mathrm{d}s$$
$$= \frac{4}{3}\eta E^* \sqrt{\beta\sigma^3} F_{3/2}(h)$$

$$h = g_N/\sigma,\ s = z/\sigma$$

$$\left.\frac{\frac{\partial p}{\partial g_N}}{p}\right|_{p_0} = \frac{3}{2\sigma} \left.\underbrace{\frac{F_{1/2}(h)}{F_{3/2}(h)}}_{=\gamma\ \approx \text{const}}\right|_{h_0(p_0)} \longrightarrow \boxed{p(g_N) = p_0 \mathrm{e}^{\frac{3}{2\sigma}\gamma(g_N - g_{N0})}}$$

$$\gamma = \gamma(h_0) \approx 1..2$$



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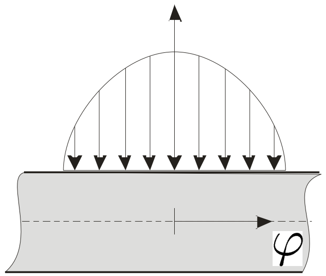
Linear contact stiffness k_p

$$p(g_{N0} + \Delta g_N) = p_0 + \left[p_0 \frac{3}{2\sigma} \gamma \right] \Delta g_N + \mathcal{O}(2)$$

p_0 - steady-state contact pressure

- pressure distribution $p_0 = p_0(\varphi)$
- usually $\gamma \approx 4/3 \ldots 6/3$

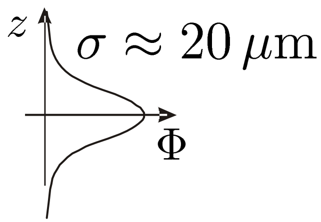


$$\boxed{k_p(\varphi, \sigma) \approx p_0(\varphi) \frac{5}{2\sigma}}$$

Experimental data (Sherif: *Investigation on effect of surface topography…on squeal generation*, Wear, 2004)

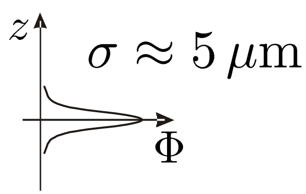
new pad

- silent -



worn

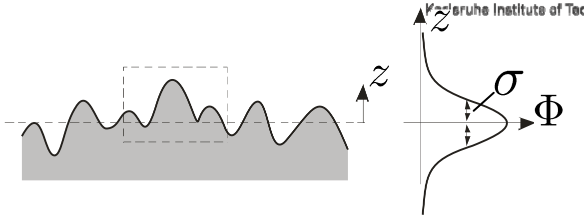
- noisy -



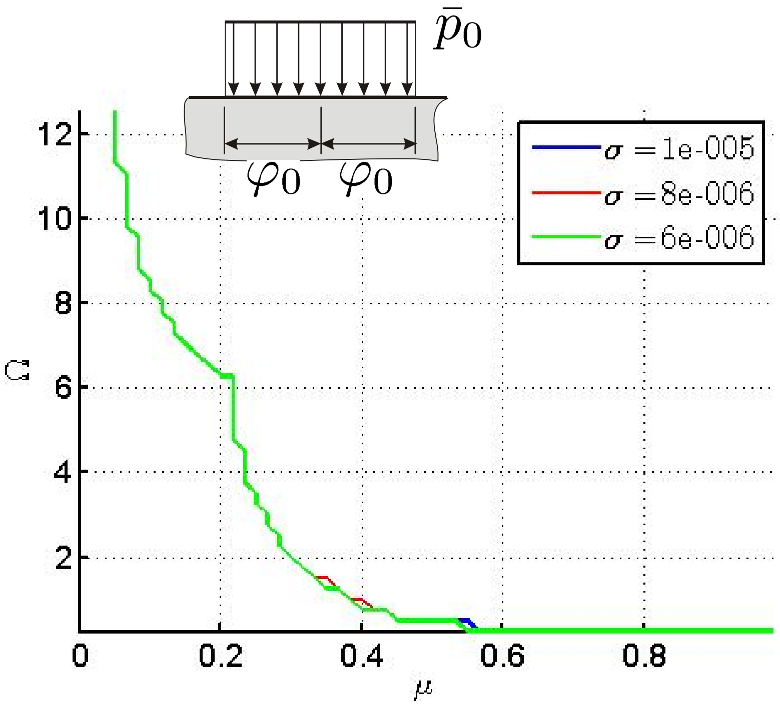
Contact pressure & surface topology

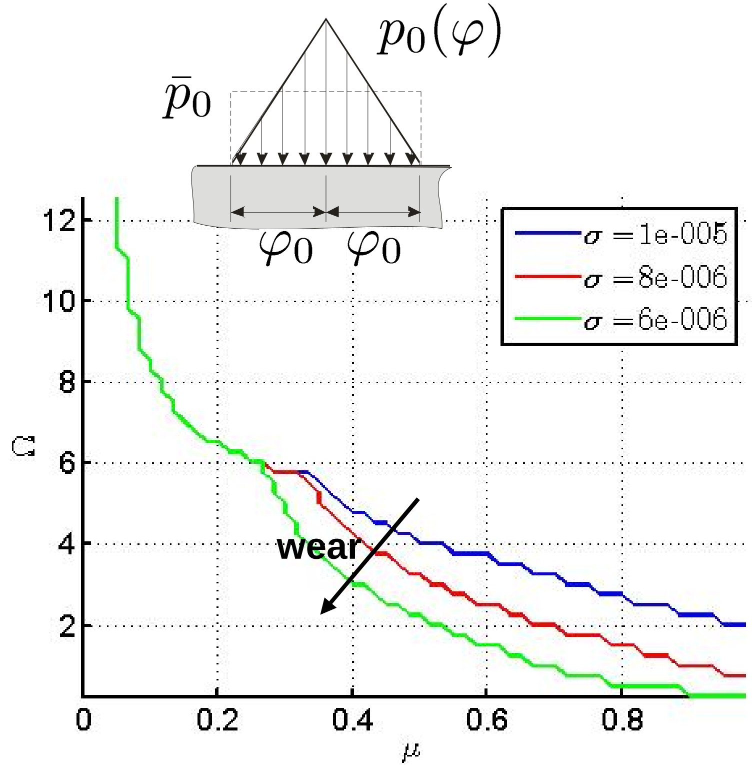
Experimental data - silent $\sigma \approx 20\,\mu\mathrm{m}$

- squeal $\sigma \approx 5\,\mu\mathrm{m}$



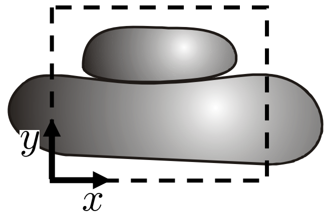
$$F_B = 100\ \mathrm{N}\ (\bar{p}_0 = 2 \cdot 10^4\ \mathrm{Pa}\ )$$





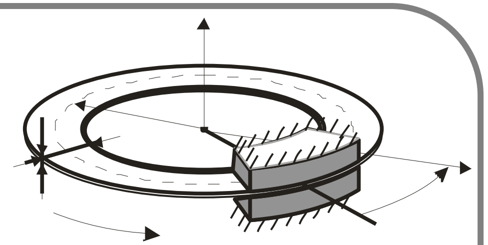
Conclusion

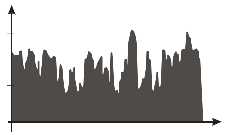
Elastic bodies in relative motion

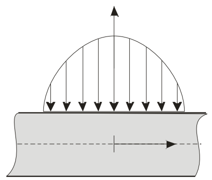


- generic form of perturbation equations
- physical meaning of the contributions

Contact mechanics and flutter-type instability



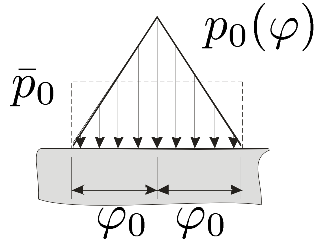




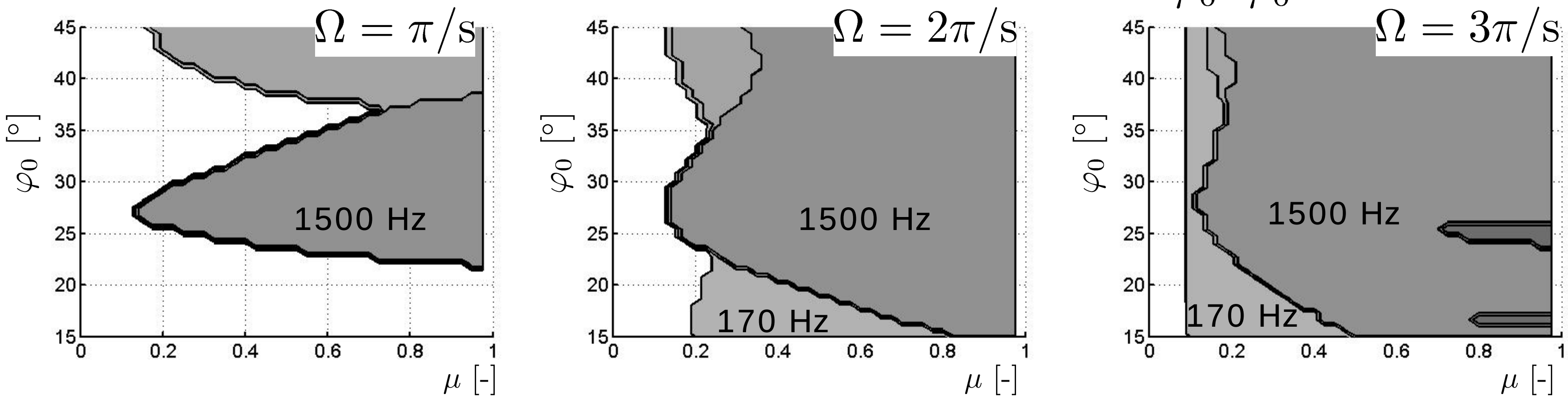
- rotating timoshenko-ring
- incr. stability for lower shear stiffness (decoupling)
- contact stiffness shows strong influence
  → tribology of contact needs to be considered
- distribution of contact pressure

Thank you for your attention!

Contact zone



$\boxed{\sigma = 20\,\mu\mathrm{m}}$ (new)



$\boxed{\sigma = 5\,\mu\mathrm{m}}$ (worn, noisy)