Karlsruhe Institute of Technology

Universität Karlsruhe (TH)
Research University • founded 1825

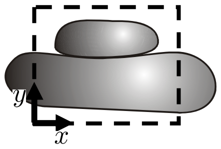
Institut für
Technische Mechanik

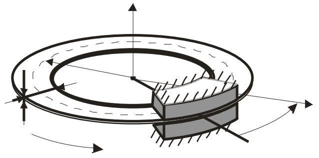
7th ISVCS, Zakopane, 2009

On self-excited vibrations due to sliding friction between moving bodies

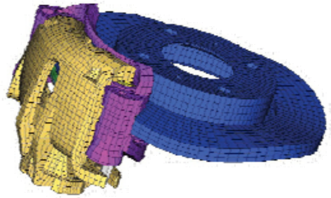
Hartmut Hetzler

- Motivation, considered class of systems
- a general formulation
- example: rotating Timoshenko annulus
- Conclusion

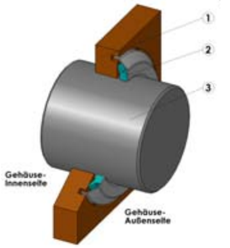




Motivation



vehicle brakes



shaft seals

band saws

grinding tools

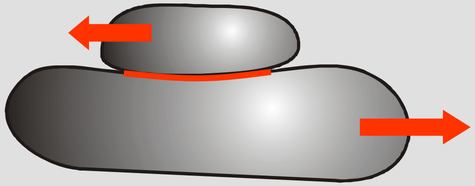
Photos: www.technik-handel.com, Bosch

usually:

- particular application
  → structural models
- specific formulation

- results generally valid?
- further effects?
- parameters

Abstract system

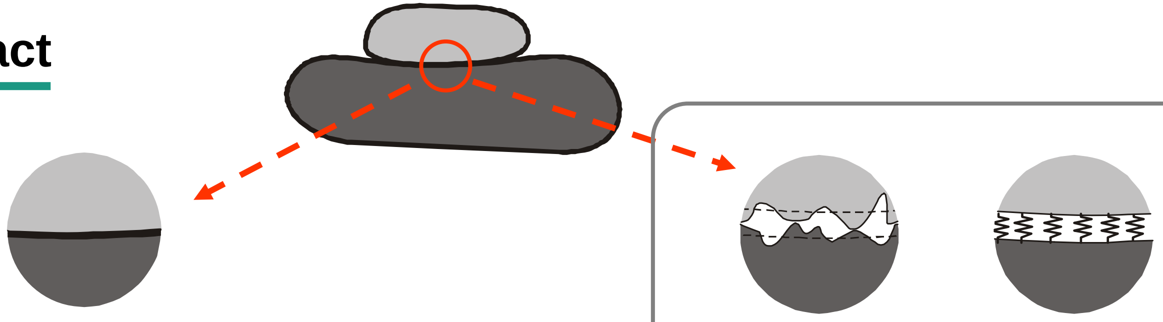


generic formulation for

- systems of continua
- relative motion
- sliding friction contacts
- spatially fixed contact zone

System Description

Normal contact



- Lagrange Multipliers:
  „ideal bodies“ → kinematic constraint

- Penalty formulation:
  „contact layer“ → contact stiffness

Hamilton‘s Principle (for open systems)

$$\boxed{\int_0^t \left\{\delta L + \delta W_{np}^* - \delta \Pi_C + \delta W_{\Gamma_C}\right\} \mathrm{dt} = 0}$$

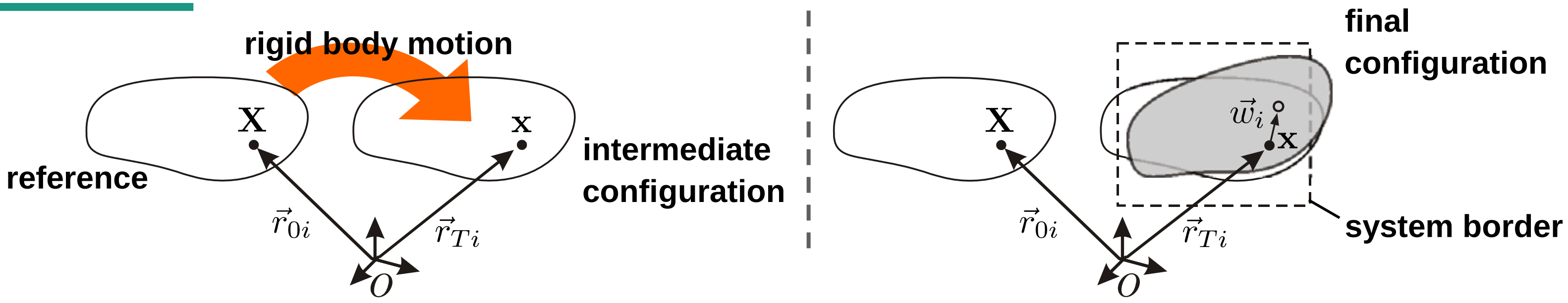
$$L = T - U$$

normal contact

tangential contact
sliding friction

Spatial frame, linearized system

Spatial description



- Eulerian coordinates of intermediate configuration $\mathbf{x} = \mathbf{x}\left(\mathbf{X}, t\right)$
- rigid body motion relates $\mathbf{x} = \mathbf{X} + \mathbf{v}_T t$
- small vibrations about transport motion → **Linearization** $\vec{r}_i = \vec{r}_{Ti} + \vec{w}_i$

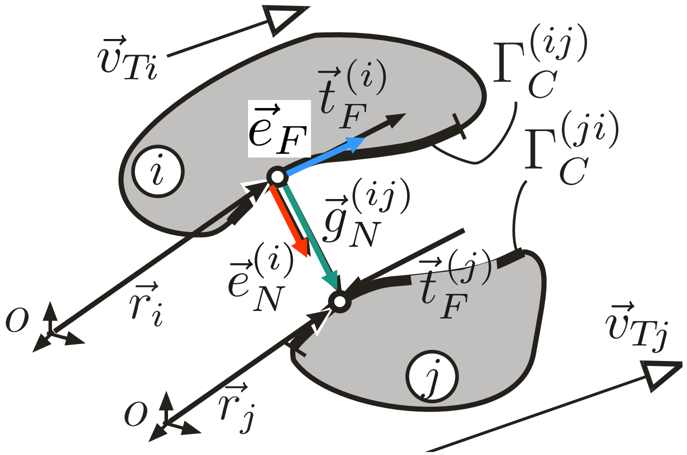
$$\boxed{0 = \sum_{i=1}^{N} \int_{\Omega_i} \delta\vec{w}_i \cdot \left(\mathcal{M}_i[\ddot{\vec{w}}_i] + \mathcal{P}_i[\dot{\vec{w}}_i] + \mathcal{Q}_i[\vec{w}_i]\right)\, \mathrm{d}v + \underbrace{\Delta\left\{\delta\Pi_C\right\} - \Delta\left\{\delta W_{\Gamma_C}\right\}}_{\textbf{linearized contact contributions}} - \Delta\left\{\delta W_{np}\right\}}$$

$$\mathcal{P}_i = \mathcal{D}_i + v_{Tij}\mathcal{G}_{ij}$$

$$\mathcal{Q}_i = \mathcal{K}_i + v_{Tij}\mathcal{N}_{ij} + v_{Tij}v_{Tik}\mathcal{K}^*_{ijk}$$

Contact

Kinematics



surface normal $\vec{e}_N^{(i)}$ ↘

gap vector ↘

$$\vec{g}^{(ij)} = \vec{r}_j - \vec{r}_i \quad = \quad (\vec{r}_{Tj} - \vec{r}_{Ti}) + (\vec{w}_j - \vec{w}_i)$$

$$= \quad \vec{g}_0^{(ij)} + \Delta\vec{g}^{(ij)}$$

normal distance

$$g_N^{(ij)} = \vec{e}_N^{(i)} \cdot \vec{g}^{(ij)} = g_{N0}^{(ij)} + \Delta g_N^{(ij)}$$

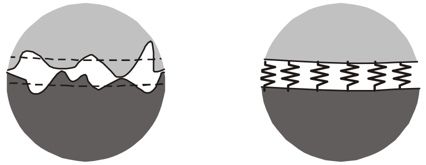
$$\Delta g_N^{(ij)} = \vec{e}_N^{(i)} \cdot \Delta\vec{g}^{(ij)}$$

$$\delta g_N^{(ij)} = \vec{e}_N^{(i)} \cdot \delta\vec{g}^{(ij)}$$

friction direction $\vec{e}_F = \vec{v}_{rel}/|\vec{v}_{rel}|$ ↗

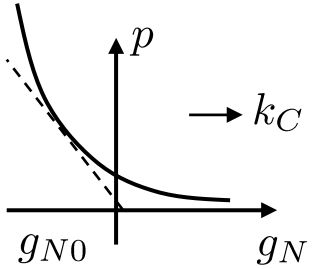
Contact

$$0=\sum_{i=1}^{N}\int_{\Omega_i}\delta\vec{w}_i\cdot\left(\mathcal{M}_i[\ddot{\vec{w}}_i]+\mathcal{P}_i[\dot{\vec{w}}_i]+\mathcal{Q}_i[\vec{w}_i]\right)\,dv+\boxed{\Delta\left\{\delta\Pi_C\right\}}-\Delta\left\{\delta W_{\Gamma_C}\right\}-\Delta\left\{\delta W_{np}\right\}$$



Normal contact

$$\delta\Pi_C=\int_{\Gamma_C}\frac{\partial\pi_C}{\partial g_N}\delta g_N\,\mathrm{da}=\int_{\Gamma_C}-p(g_N)\,\delta g_N\,\mathrm{da}$$



$$\rightarrow k_C=\left.\frac{\partial p}{\partial g_N}\right|_0$$

$$\delta\Pi_C\approx-\int_{\Gamma_C}\delta g_N\left(p_0-k_C\Delta g_N\right)\,\mathrm{da}$$

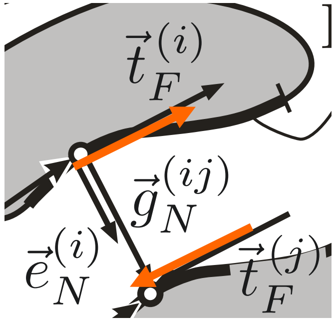
$$\Delta\left\{\delta\Pi_C\right\}=\int_{\Gamma_C^{(ij)}}k_C\delta g_N\Delta g_N\,\mathrm{da}$$

$$\delta g_N=\delta\vec{g}\cdot\vec{e}_N \qquad \Delta g_N=\vec{e}_N\cdot\Delta\vec{g}$$

$$\begin{aligned}\Delta\left\{\delta\Pi_C\right\}&=\int_{\Gamma_C^{(ij)}}k_C\left(\delta\vec{g}\cdot\vec{e}_N\right)\left(\vec{e}_N\cdot\Delta\vec{g}\right)\,\mathrm{da}\\&=\int_{\Gamma_C^{(ij)}}\delta\vec{g}\left[k_C\vec{e}_N\otimes\vec{e}_N\right]_0\Delta\vec{g}\,\mathrm{da}=\int_{\Gamma_C^{(ij)}}\delta\vec{g}\cdot\mathcal{C}[\Delta\vec{g}]\,\mathrm{da}\end{aligned}$$

Contact

$$0=\sum_{i=1}^{N}\int_{\Omega_i}\delta\vec{w}_i\cdot\left(\mathcal{M}_i[\ddot{\vec{w}}_i]+\mathcal{P}_i[\dot{\vec{w}}_i]+\mathcal{Q}_i[\vec{w}_i]\right)dv+\Delta\{\delta\Pi_C\}\boxed{-\Delta\{\delta W_{\Gamma_C}\}}-\Delta\{\delta W_{np}\}$$



Friction

$$\vec{t}_F^{\,(i)}=-\vec{t}_F^{\,(j)}=\mu p_N\vec{e}_F$$

$$\delta W_C=-\int_{\Gamma_C^{(ij)}}[\delta\vec{r}_j-\delta\vec{r}_i]\cdot\mu p_N\vec{e}_F\,\mathrm{da}=-\int_{\Gamma_C^{(ij)}}\delta\vec{g}\cdot\underline{\mu p_N\vec{e}_F}\,\mathrm{da}$$

Linearization, $\mu=$const

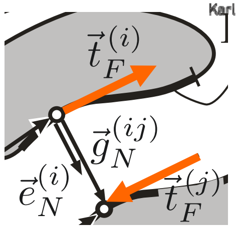
$$-\Delta\{\delta W_C\}=\int_{\Gamma_C^{(ij)}}\delta\vec{g}\cdot\overline{\left([\mu p_N]_0\Delta\vec{e}_F+[\mu\vec{e}_F]_0\Delta p_N\right)}\,\mathrm{da}$$

$$\Delta\vec{e}_F\approx\frac{1}{v_{rel,0}}\Delta\vec{v}_{rel}=\frac{1}{v_{rel,0}}\left[\Delta\dot{\vec{g}}+\mathbf{v}_{Tj}^{\top}\frac{\partial\vec{w}_j}{\partial\mathbf{x}}-\mathbf{v}_{Ti}^{\top}\frac{\partial\vec{w}_i}{\partial\mathbf{x}}\right]$$

$$\Delta p=-k_C\Delta g_N=-k_C\vec{e}_N\cdot\vec{g}$$

Contact

$$0 = \sum_{i=1}^{N} \int_{\Omega_i} \delta\vec{w}_i \cdot \Big(\mathcal{M}_i[\ddot{\vec{w}}_i] + \mathcal{P}_i[\dot{\vec{w}}_i] + \mathcal{Q}_i[\vec{w}_i]\Big)\, dv + \Delta\{\delta\Pi_C\} \boxed{- \Delta\{\delta W_{\Gamma_C}\}} - \Delta\{\delta W_{np}\}$$



Friction […]

orientation of friction vector

contact pressure

$$= \int_{\Gamma_C^{(ij)}} \delta\vec{g}\cdot\left(\left[\frac{\mu p_N}{v_{rel}}\right]_0 \Delta\dot{\vec{g}} + \left[\frac{\mu p_N}{v_{rel}}\right]_0 \left(\mathbf{v}_{Tj}^{\top}\frac{\partial\vec{w}_j}{\partial\mathbf{x}} - \mathbf{v}_{Ti}^{\top}\frac{\partial\vec{w}_i}{\partial\mathbf{x}}\right) - \left[\mu k_C(\vec{e}_F\otimes\vec{e}_N)\right]_0 \Delta\vec{g}\right) \mathrm{da}$$

$$= \int_{\Gamma_C^{(ij)}} \delta\vec{g}\cdot\Big(\mathcal{R}_1[\Delta\dot{\vec{g}}] + \mathcal{R}_2[\vec{w}_1,\vec{w}_2] + \mathcal{R}_3[\Delta\vec{g}]\Big)\,\mathrm{da}$$

damping

$\mathcal{R}_1 = \mathcal{R}_1^{\top}$

$\mathcal{R}_1 \geq 0$

stiffness

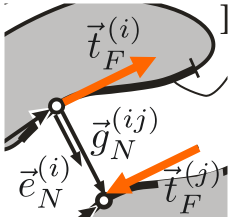
$\mathcal{R}_2 \neq \mathcal{R}_2^{\top}$

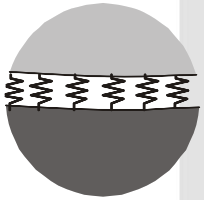
stiffness

$\mathcal{R}_3 \neq \mathcal{R}_3^{\top}$

Contact contributions

$$0=\sum_{i=1}^{N}\int_{\Omega_i}\delta\vec{w}_i\cdot\left(\mathcal{M}_i[\ddot{\vec{w}}_i]+\mathcal{P}_i[\dot{\vec{w}}_i]+\mathcal{Q}_i[\vec{w}_i]\right)\,dv+\boxed{\Delta\left\{\delta\Pi_C\right\}-\Delta\left\{\delta W_{\Gamma_C}\right\}}-\Delta\left\{\delta W_{np}\right\}$$





$$\Delta\left\{\delta\Pi_C\right\}-\Delta\left\{\delta W_{\Gamma_C}\right\}$$

$$=\int_{\Gamma_C^{(ij)}}\delta\vec{g}\cdot\Big(\ \left[k_C(\vec{e}_N\otimes\vec{e}_N)\right]_0\Delta\vec{g}$$

← **normal contact**

frictional contact

$$+\left[\frac{\mu p_N}{v_{rel}}\right]_0\Delta\dot{\vec{g}}+\left[\frac{\mu p_N}{v_{rel}}\right]_0\left(\mathbf{v}_{Tj}^{\top}\frac{\partial\vec{w}_j}{\partial\mathbf{x}}-\mathbf{v}_{Ti}^{\top}\frac{\partial\vec{w}_i}{\partial\mathbf{x}}\right)-\left[\mu k_C(\vec{e}_F\otimes\vec{e}_N)\right]_0\Delta\vec{g}\Big)\,\mathrm{da}$$

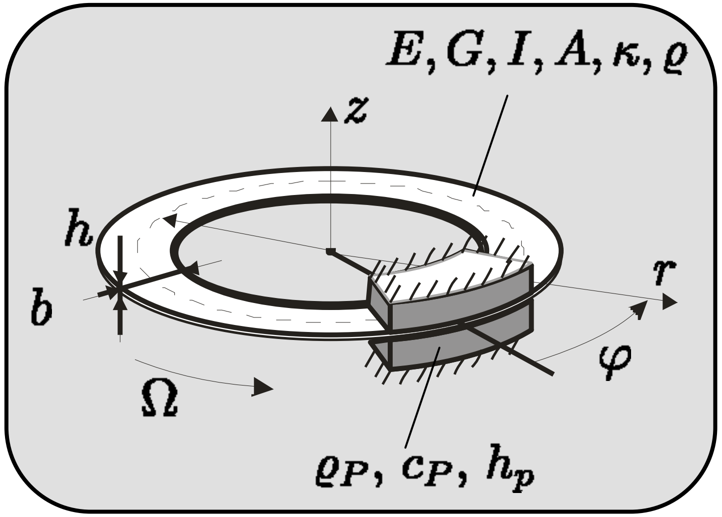
→ **normal contact**
- stiffness
- symmetric, pos. semidefinite

→ **friction**
- damping and „stiffness“
- damping: symmetric, positive semidefinite grows with 1/v
- stiffness: non-symmetric

parameters $\mu,\ k_C,\ p_N,\ v_{rel},\ \Gamma_C$

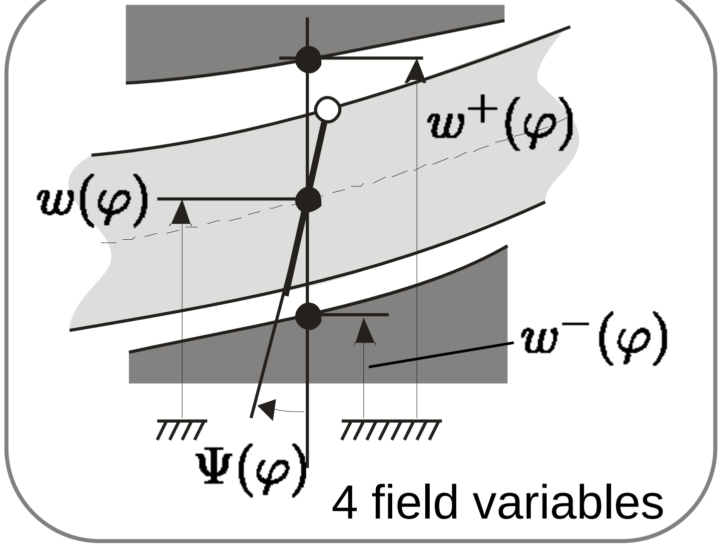
- **no discretization**
- **no structural model**

Example: rotating Timoshenko ring



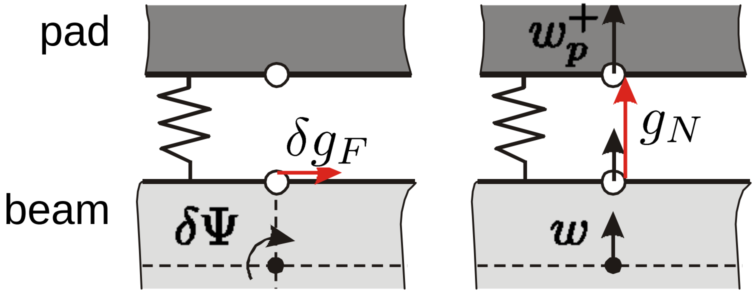
- rotating circular Timoshenko beam
- friction pads as Winkler foundation
- Eulerian description
- simple model for brake squeal

data: $\kappa = 5/6,\ R = 0.12\mathrm{m},\ h = 0.01\mathrm{m},\ b = 0.1\mathrm{m},\ \varphi_0 = \pi/8,$
$c_P = 8\cdot 10^8\mathrm{Pa/m},\ h_p = 0.02\mathrm{m},\ k_p = 5\cdot 10^{10}\ \mathrm{Pa/m},$
$E = 2.1\cdot 10^{11}\mathrm{Pa},\ \nu = 0.33,\ \varrho = 7800\mathrm{kg/m}^3$



4 field variables

$$=\sum_{(ij)}^{n_C}\int_{\Gamma_C^{(ij)}}\delta\vec{g}\cdot\big(\ldots+\underbrace{\mu k_C\left(\vec{e}_F\otimes\vec{e}_N\right)\Delta\vec{g}}_{\delta g_F\,\mu\,(k_p\Delta g_N)}\big)\ \mathrm{d}a$$



Stability of steady-state

$$0 = \sum_{i=1}^{N} \int_{\Omega_i} \delta\vec{w}_i \cdot \left(\mathcal{M}_i[\ddot{\vec{w}}_i] + \mathcal{P}_i[\dot{\vec{w}}_i] + \mathcal{Q}_i[\vec{w}_i]\right) \mathrm{d}v + \Delta\{\delta\Pi_C\} - \Delta\{\delta W_{\Gamma_C}\} - \Delta\{\delta W_{np}\}$$

discretization

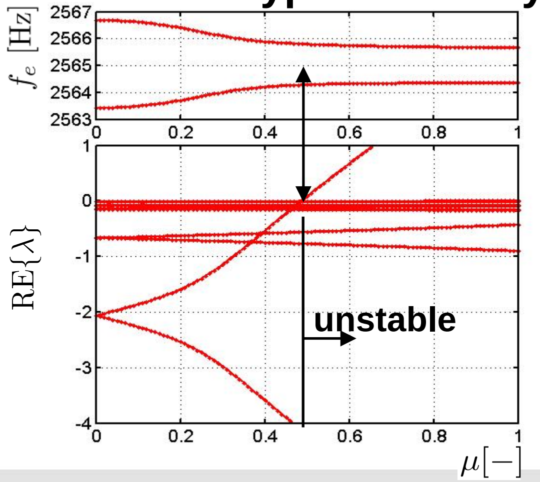
$$\Delta\{\delta\Pi_C\} - \Delta\{\delta W_{\Gamma_C}\}$$

$$= \sum_{(ij)}^{n_C} \int_{\Gamma_C^{(ij)}} \delta\vec{g} \cdot \left(k_C\,(\vec{e}_N \otimes \vec{e}_N)\,\Delta\vec{g} \;+\; \frac{\mu p_0}{v_T}\Delta\dot{\vec{g}} + \overset{\approx 0}{\cancel{\mu p_0 \frac{\partial}{\partial x}}}\Delta\vec{g} + \mu k_C\,(\vec{e}_F \otimes \vec{e}_N)\,\Delta\vec{g} \right) \mathrm{d}a$$

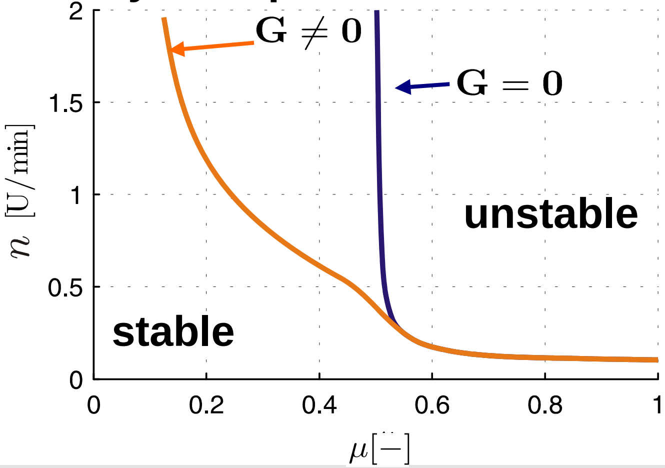
$$\mathbf{M}\ddot{\mathbf{q}} + \left(\Omega\mathbf{G} + \mathbf{D}_S + \frac{\mu p_0}{\Omega}\mathbf{R}_1\right)\dot{\mathbf{q}} + \left(\mathbf{K}_S + k_C\mathbf{K}_N + \mu k_C h\mathbf{R}_3\right)\mathbf{q} = \mathbf{0}$$

damped gyroscopic circulatory system

Flutter-type instability



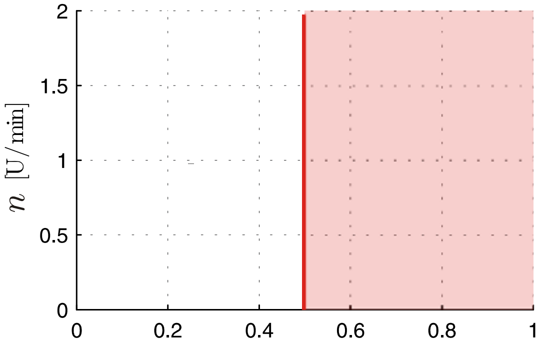
Gyroscopic influence



Stability of trivial solution

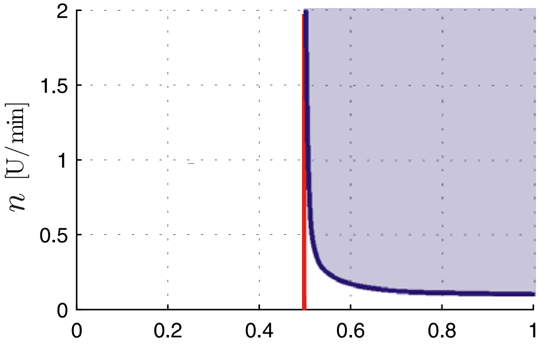
Influence of $\mathbf{R}_1$ and $\mathbf{G}$

$$\mathbf{M}\ddot{\mathbf{q}} + \Big(\qquad + \mathbf{D}_S \qquad \Big)\,\dot{\mathbf{q}} + (\mathbf{K} + \mu k_C h \mathbf{R}_3)\,\mathbf{q} = \mathbf{0}$$



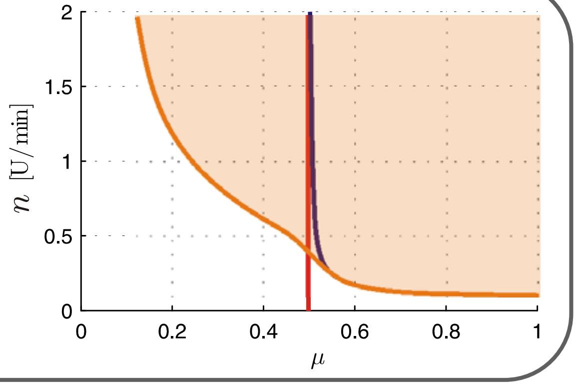
$$\mathbf{M}\ddot{\mathbf{q}} + \Big(\qquad + \mathbf{D}_S + \frac{\mu p_0}{\Omega}\mathbf{R}_1 \Big)\,\dot{\mathbf{q}} + (\mathbf{K} + \mu k_C h \mathbf{R}_3)\,\mathbf{q} = \mathbf{0}$$

- friction contribution to damping important at low relative speeds



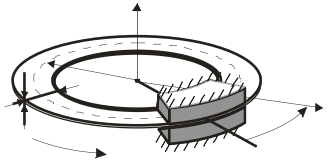
$$\mathbf{M}\ddot{\mathbf{q}} + \Big(\Omega\mathbf{G} + \mathbf{D}_S + \frac{\mu p_0}{\Omega}\mathbf{R}_1 \Big)\,\dot{\mathbf{q}} + (\mathbf{K} + \mu k_C h \mathbf{R}_3)\,\mathbf{q} = \mathbf{0}$$

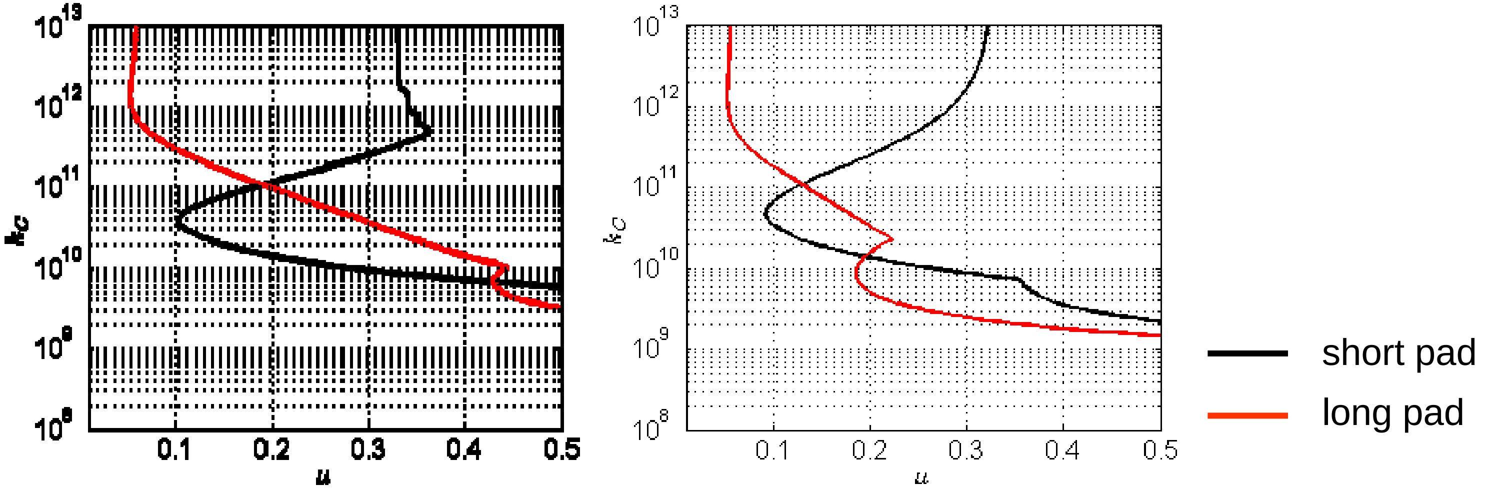
- gyroscopic terms have significant effect
- Theorem of Thomson&Tait does not apply to circulatory systems!



Contact stiffness

$$\mathbf{M}\ddot{\mathbf{q}} + \left(\Omega\mathbf{G} + \mathbf{D}_S + \frac{\mu p_0}{\Omega}\mathbf{R}_1\right)\dot{\mathbf{q}} + \left(\mathbf{K} + \mu k_C h \mathbf{R}_3\right)\mathbf{q} = \mathbf{0}$$

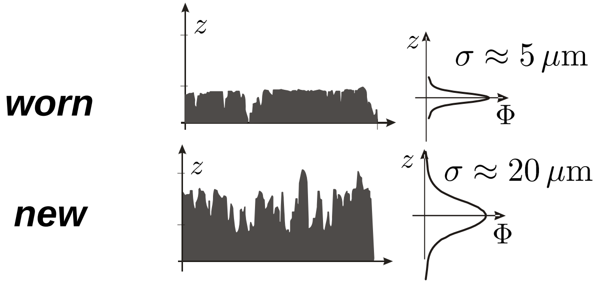




Constitutive contact model

- Greenwood&Williamson
- surface statistics

$$\boxed{k_C(\sigma) \approx p_0 \frac{5}{2\sigma}}$$

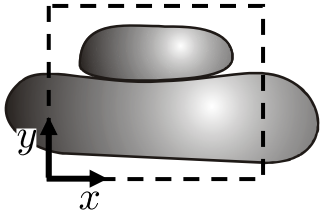


(Sherif: *Investigation on effect of surface topography…on squeal generation*, Wear, 2004)

Conclusion

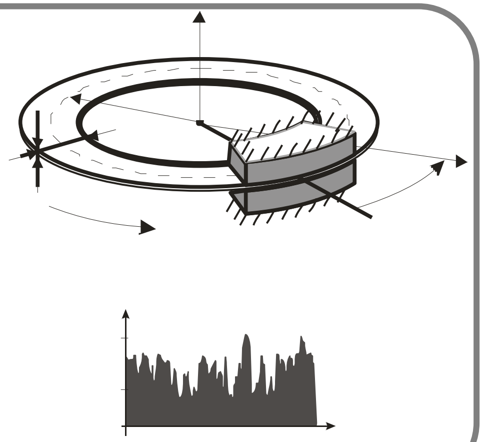
Abstract problem

- generic form of perturbation equations
- physical meaning of the contributions
- no discretization / no structural model
- systematic way to formulate contributions



Example: moving Timoshenko-Ring

- rotating timoshenko-ring
- strong influence of transport motion and „friction damping“
- contact stiffness shows strong influence
  → microcmechanics of contact need to be considered



Thank you for your attention!