Low frequency brake vibrations

- experiment, pseudo-phasespace embedding, modelling -

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Chicago

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Institute for
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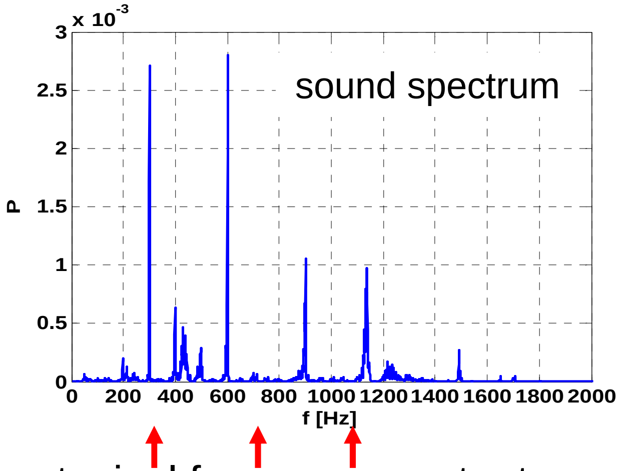
Phenomenon

- Disc-Brake Noise: *groan, muh*
  basic frequency: ~300 Hz
- very low driving speeds
  (→ low ambient noise!)
- mostly cars with automatic-gearshift

n=1 U/min, F_B=1500N

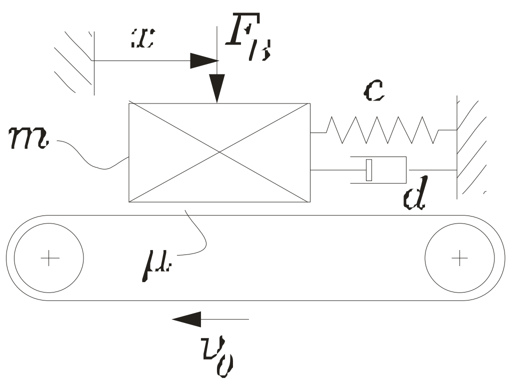


typical frequency contents

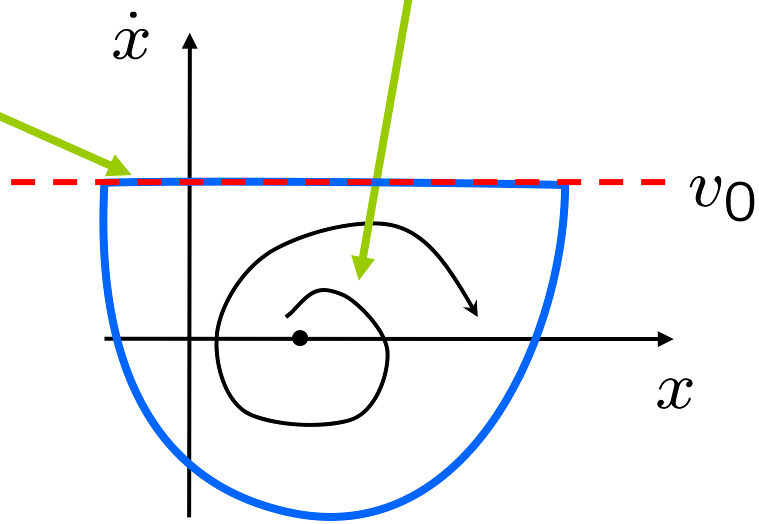
<u>Conventional explanation</u> self-excitation due to friction

onset: - friction characteristic with negative slope
- flutter through eigenvalue coupling

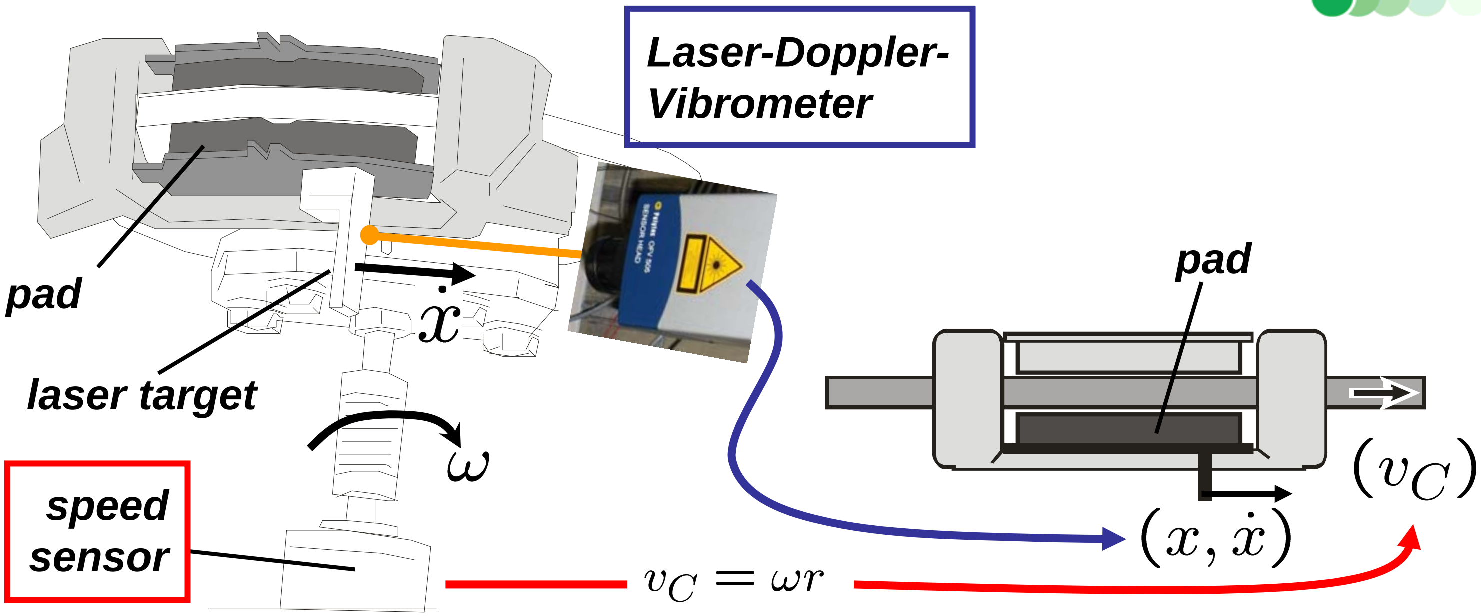
limit-cycle: stick-slip mechanism



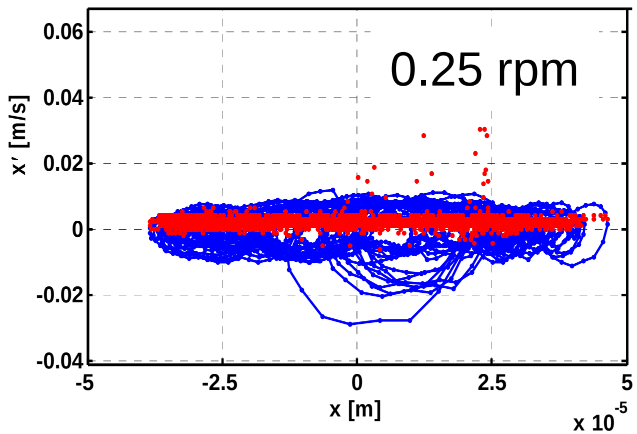
example.: Stick-Slip - Oscillator

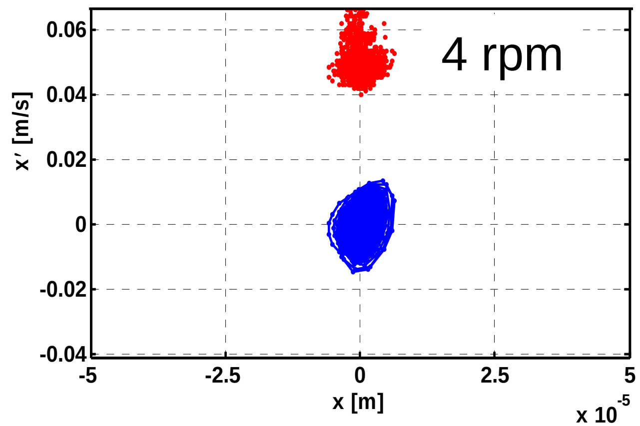


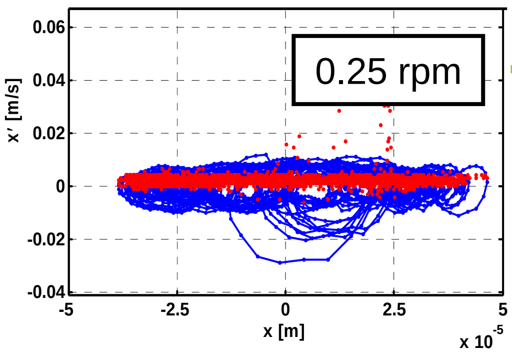
example.: typical Stick-Slip – limit Cycle

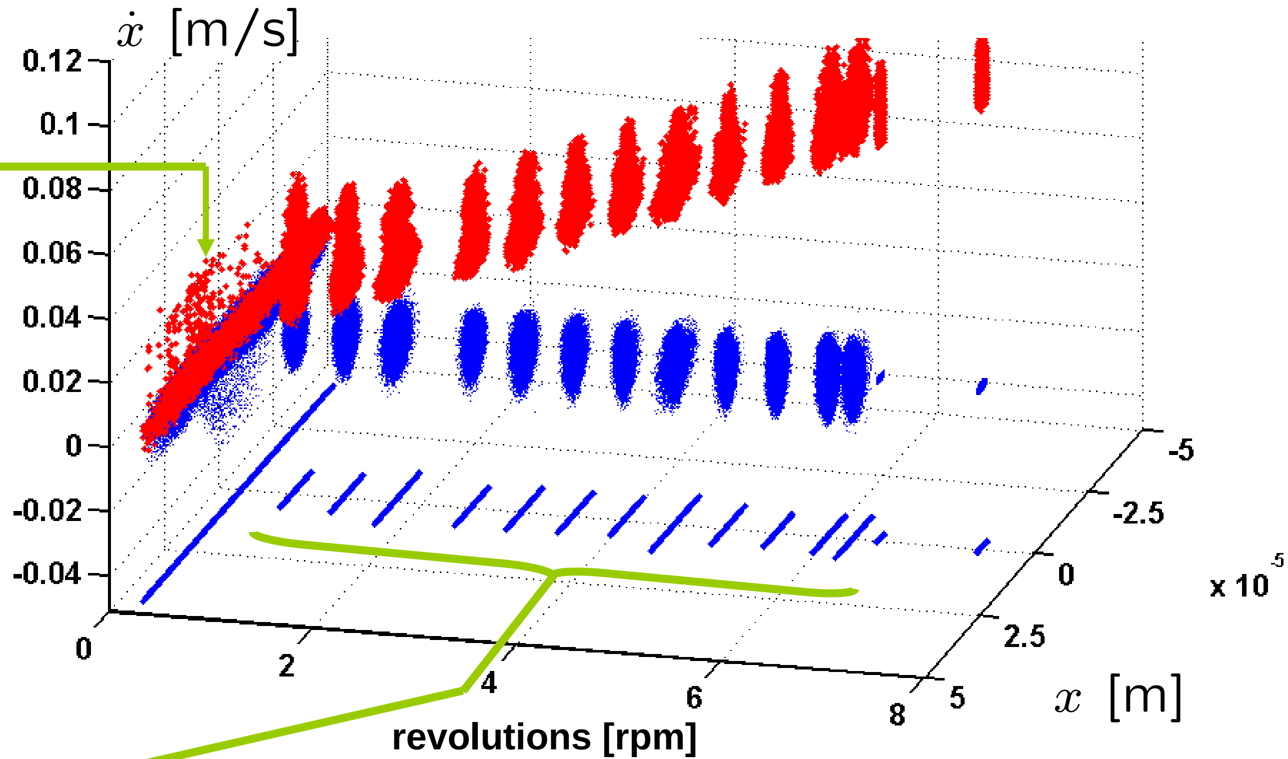


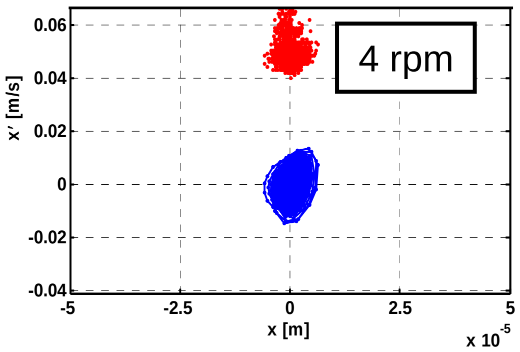
disc surface (speed v_C) **pad** (displacement x , speed $\dot{x}$)



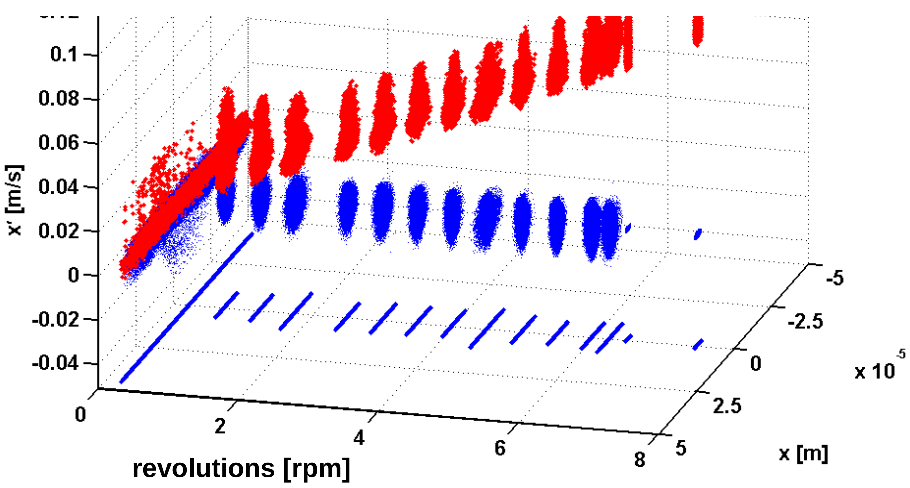


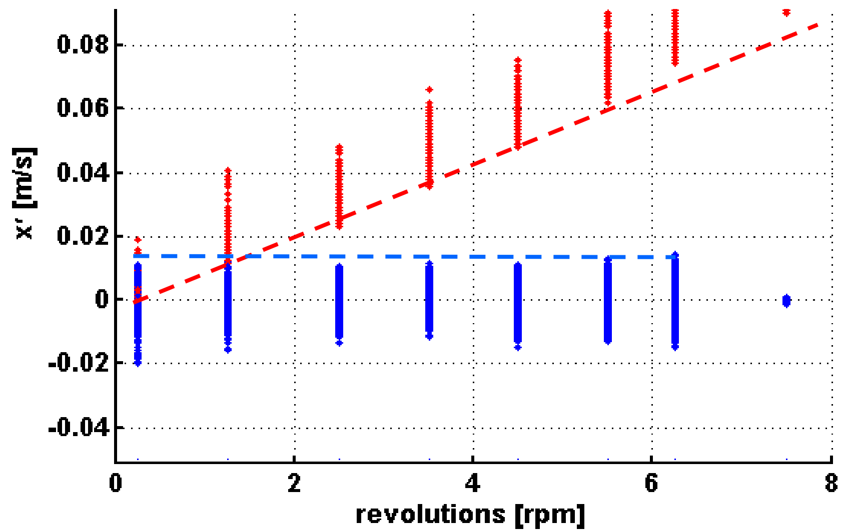




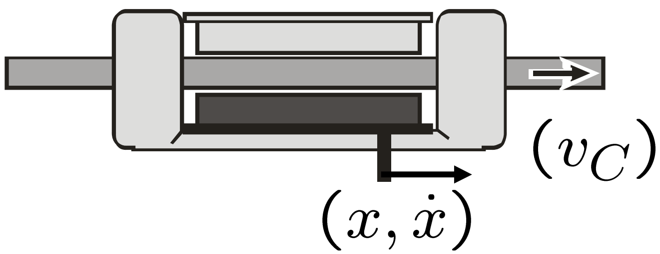


- **two types of oscillation**
  1) large amplitudes, vanishing relative speed
  2) smaller amplitudes, non-vanishing relative speed
- **oscillations up to ~6.5 rpm**
- **acoustic impression unchanged** (almost)





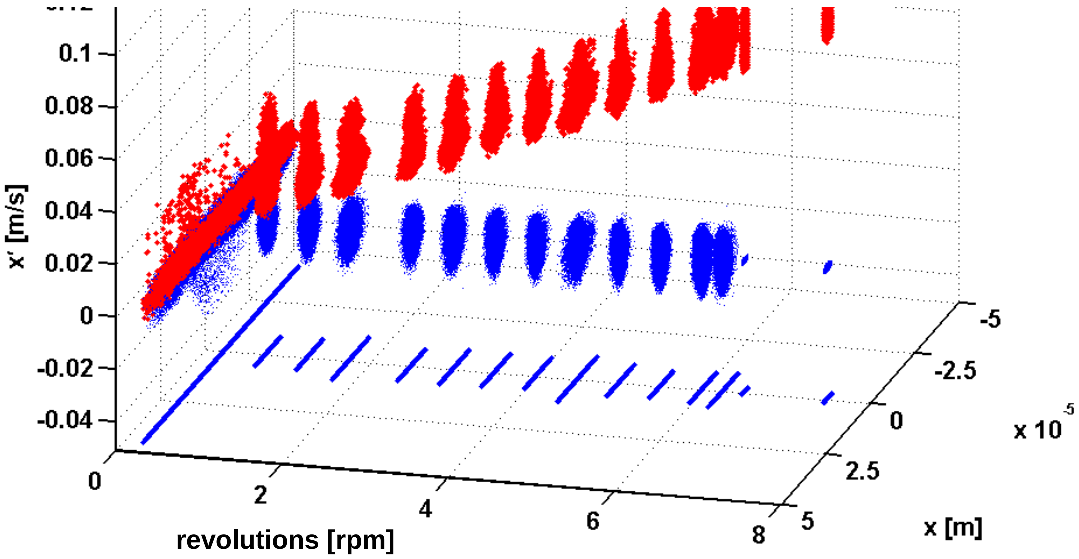
Scaling behaviour



disc surface: $v_C \propto$ rev.-speed

pad: $\dot{x}_{max} =$ const

… stiction between pad and disc?



What is this?
How to model this?

Questions

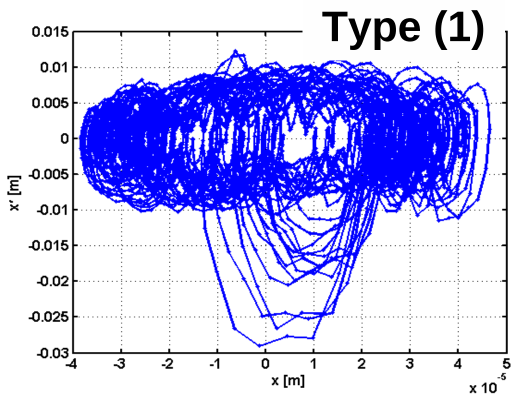
- Are there Limit-Cycles or Periodic Attractors ?
- What dimension needs a state-space to contain them?
- What mechanism drives these oscillations?
  Is it really a „stick-slip" – mechanism?

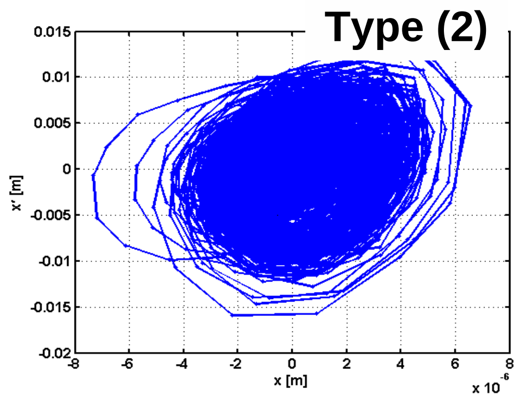
First try…

Pad‘s
displacement & speed
$(x, \dot{x})$

stationary data

duration: ~1sec



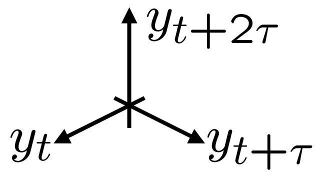


→ **Periodic Attractor?** recognizable nothing recognizable

Idea: use phase-space reconstruction to see if there‘s an attractor

→ spanning a pseudo phase-space with delayed time series

$$y_t, y_{t+\tau}, y_{t+2\tau}, \cdots, y_{t+(n-1)\tau}$$



Issues: τ - delay, lag n - dimension → paper !

strategy:

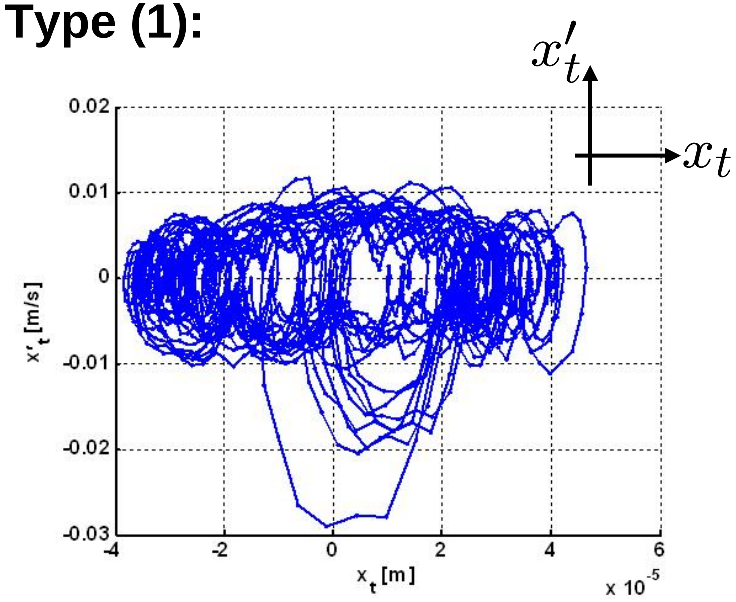
classical phase-diagram, unfolded by means of delayed time series

x_t - displacement data time series

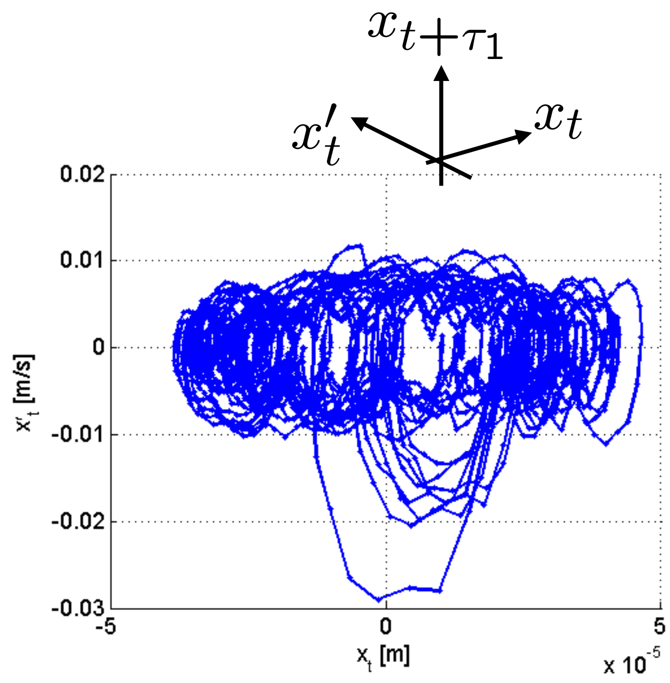
x'_t - velocity data

$x_{t+\tau_1}$ - delayed displacement time series

Type (1):

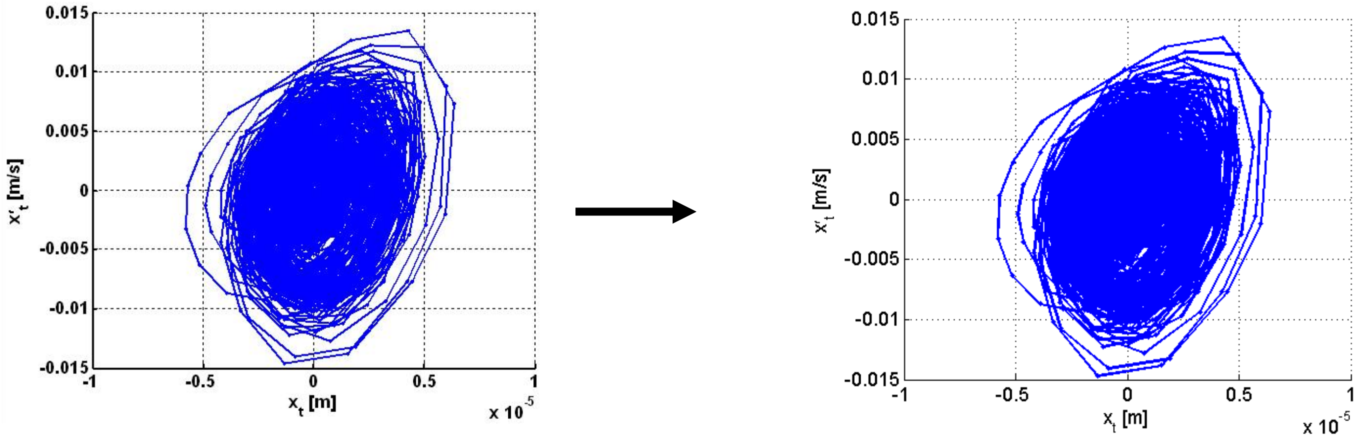


unfolding

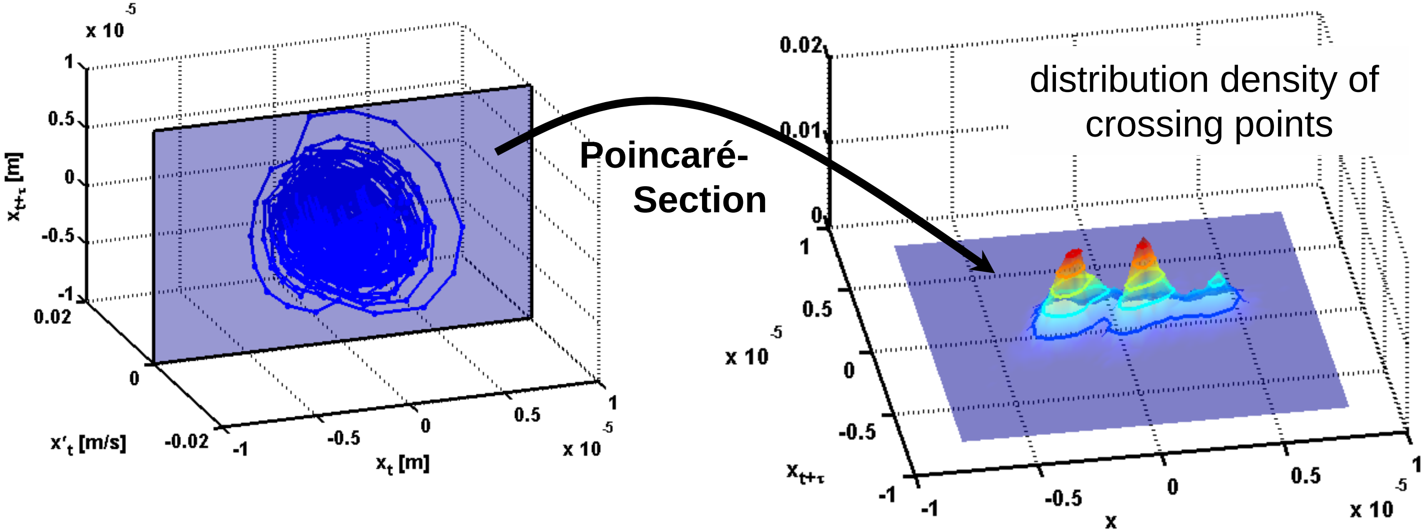


periodicity already visible in 3d-phase-space!

Type (2):



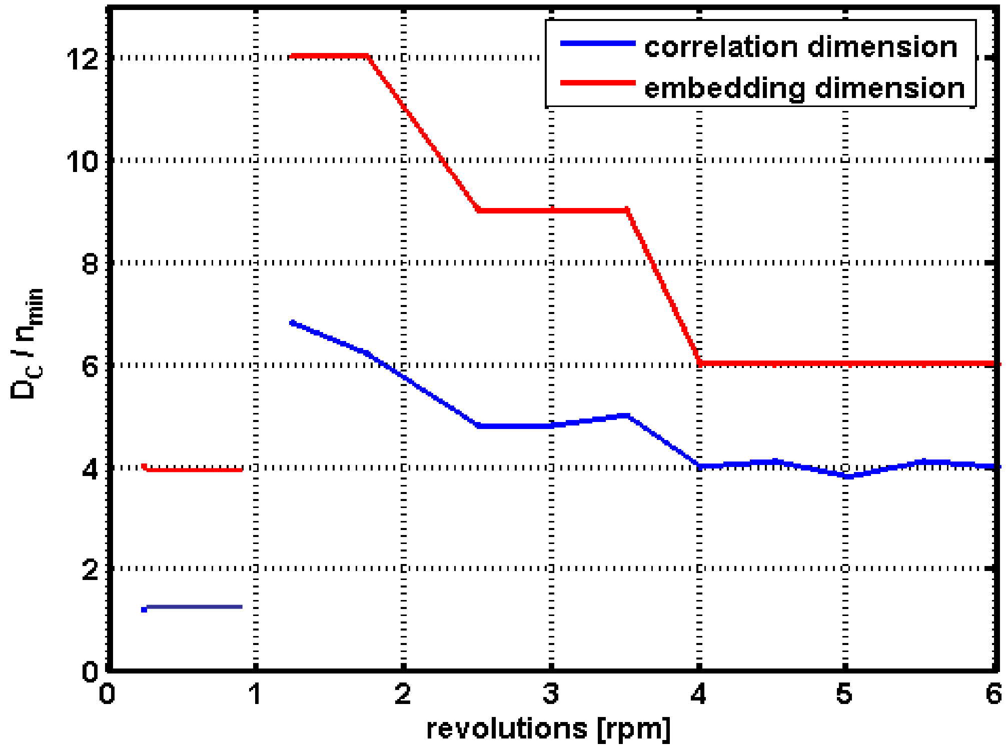
straight-forward attempt: - no periodicity visible in 3d-phase-space
- but: seems to have a structure



- not only random
- structured
- periodic

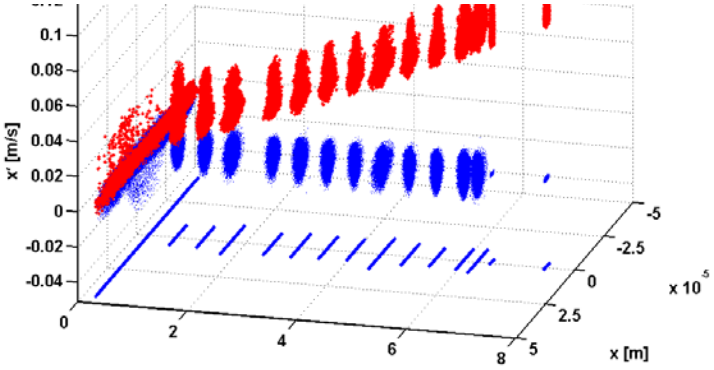
Embedding Dimension n : minimal phase-space dimension

Correlation Dimension $D_C(n, r)$: (local) attractor dimension



partially high-dimensional phase-space dimension required

again:
clearly two-fold behavor



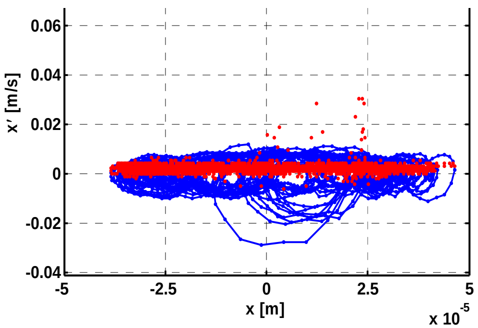
2 types of motion

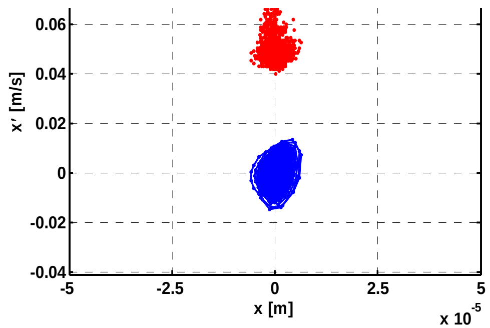
type (1)

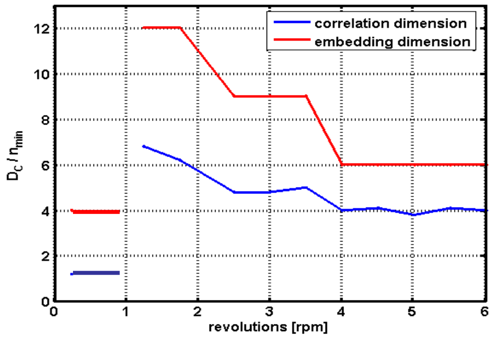
- extremely low speeds
- relative speed may vanish
- large amplitudes
- phase-space dimension $n_{min} \approx 4$

type (2)

- moderate speeds
- relative speed does not vanish
  → *stick-slip unlikely*
- smaller amplitudes , independent of driving speed
- phase-space dimension $n_{min} \approx 6 - 12$

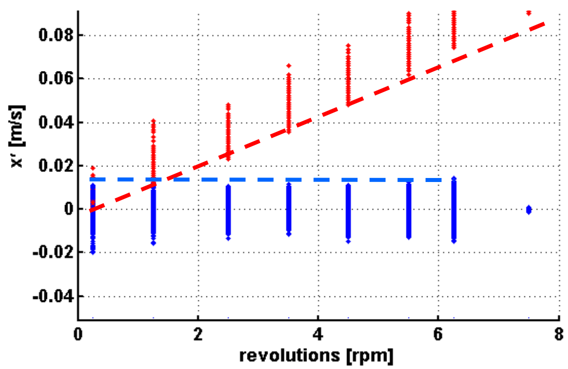






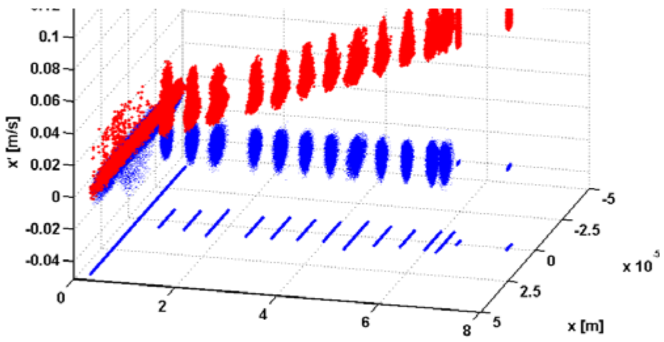
Phase-space of finite dimension

- partially high-dimensional phase-space required



no scaling of amplitudes with driving speed

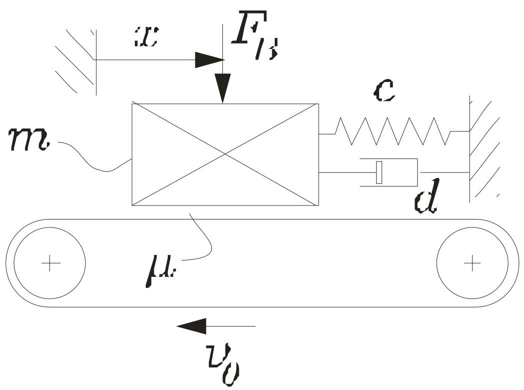
may this be explained by a stick-slip mechanism ?

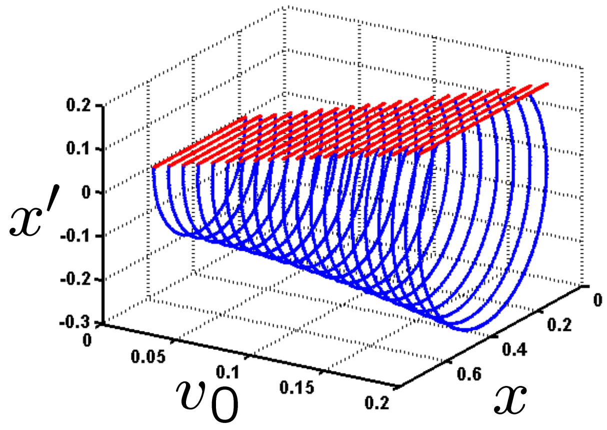


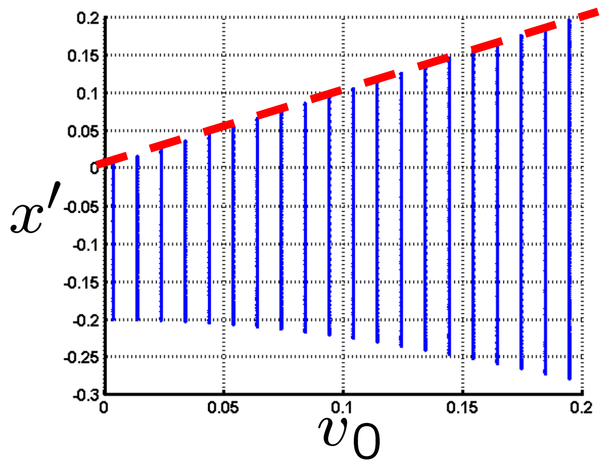
May the stick-slip mechanism produce this scaling behavior?

„mass-on-a-conveyer“-modell

- spring-damper-oscillator
- stick-slip - friction



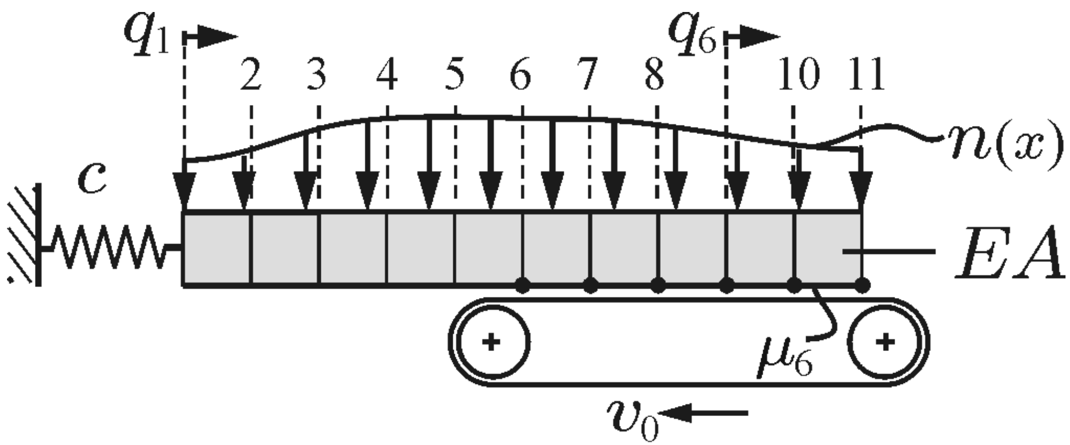




max. speed scales linearly with the driving speed !

What about elastic properties?

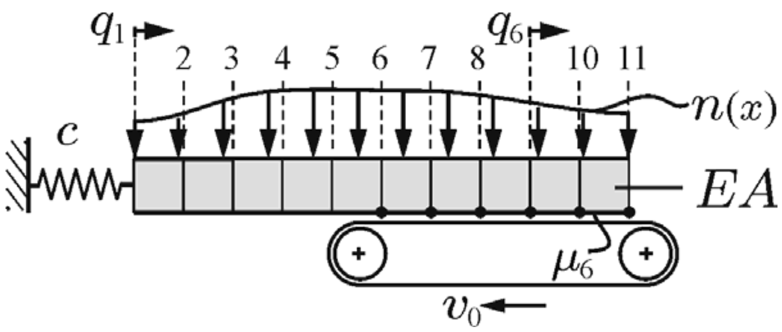
elastic beam on a conveyor

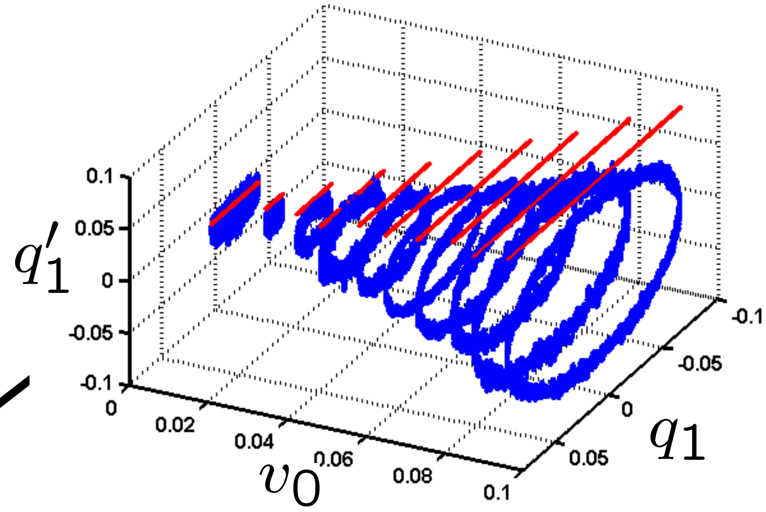


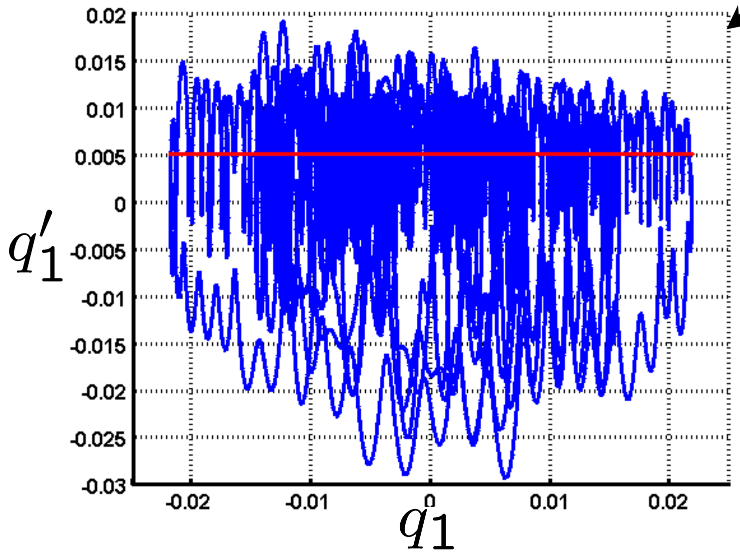
- linear elastic beam (EA)
- elastic coupling to environment
- FE-discretization
- stick-slip friction at the nodes
- Lagrangian Multipliers / event-driven sim.

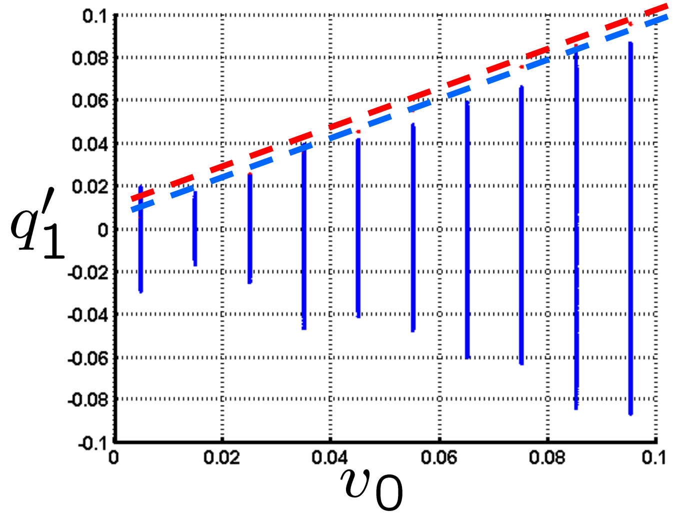
$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{C}\mathbf{q} = \mathbf{f}_S + \mathbf{G}^T\lambda$$

elastic properties…







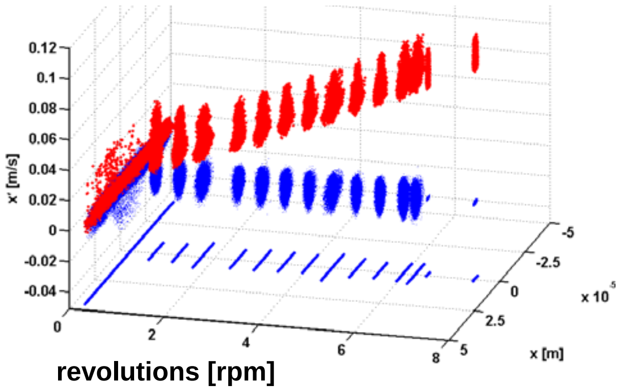


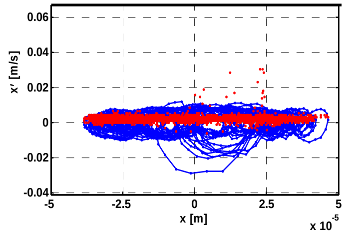
again: linear scaling !

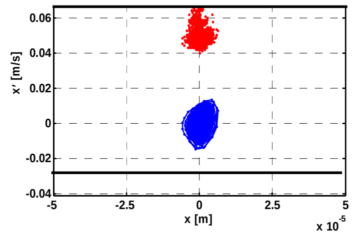
Initial Questions: What is this groaning noise? How to model it?

Experiment:

- it‘s twofold
  - -> type (1): „stick-slip“
  - -> type (2): no stiction
- unexpected scaling behavior (it doesn‘t scale!)

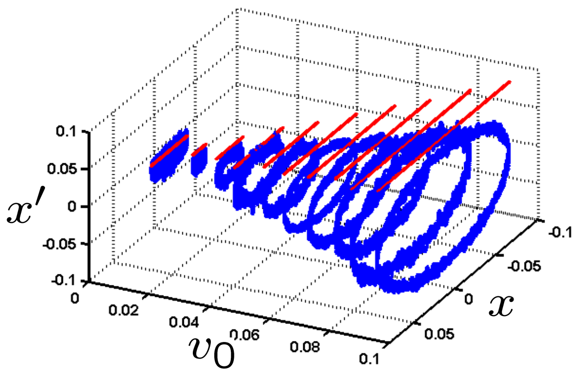


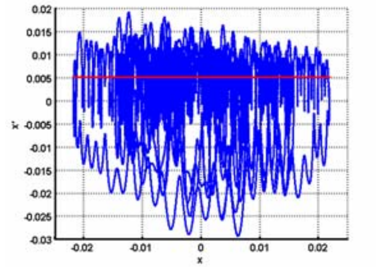




Simulation:

- stick-slip oscillators:
  - don‘t show scaling from exp. !
- stick-slip with elastic properties:
  - → simulation of type (1)





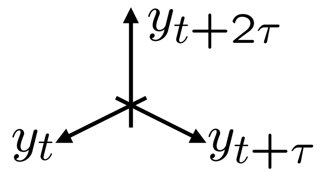
Open Question: What is this „Type 2“ – Oscillation ?

_ .

Thank you

for your attention !

Phase-space spanned by delayed time-series



$$y_t, y_{t+\tau}, y_{t+2\tau}, \ldots, y_{t+(n-1)\tau}$$

τ - delay, lag n - dimension

objects in the reconstructed pseudo-phase-space will have the same topological properties than in the (unknown) genuine phase-space

→ limit cycles: closed curve, no crossing, …

Issues: choice of lag τ and dimension n

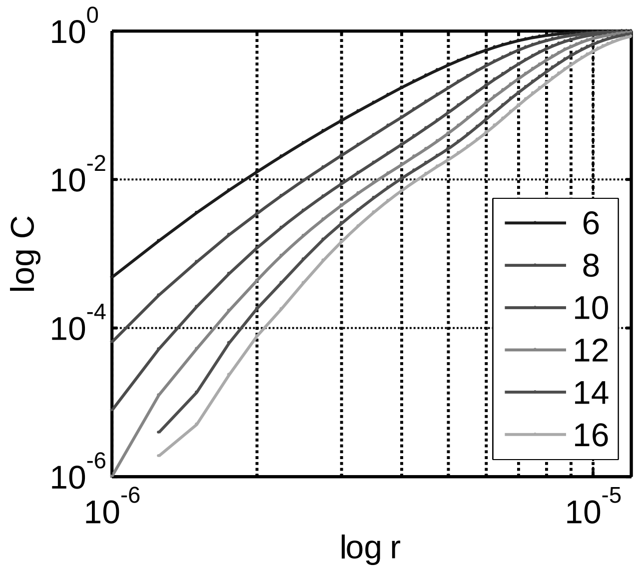
Choice of lag τ: delay vectors should be mutually „independent“, „perpendicular“

- **Average Mutual Information** $I_{y_t y_{t+\tau}}(\tau)$ as measure of statistical dependency

experiments with simulated stick-slip data showed GOOD performance !

with: Correlation Sum

$$C(n,r) = \frac{2}{N(N-1)} \sum_i^N \sum_{j=i}^N \Theta \left[r - ||\mathbf{s_i} - \mathbf{s_j}|| \right]$$

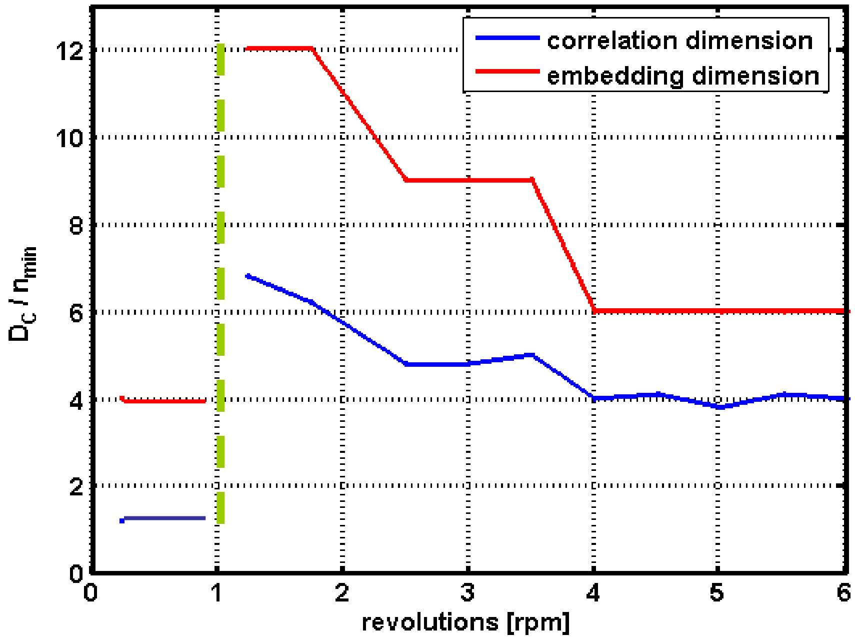


Invariant Measure: Correlation Dimension

$$D_C(n,r) = \frac{\partial \log C(n,r)}{\partial \log r}$$

Idea: minimal dimension n_{min}
reached, if $D_C(n,r)$
does not change with changing n

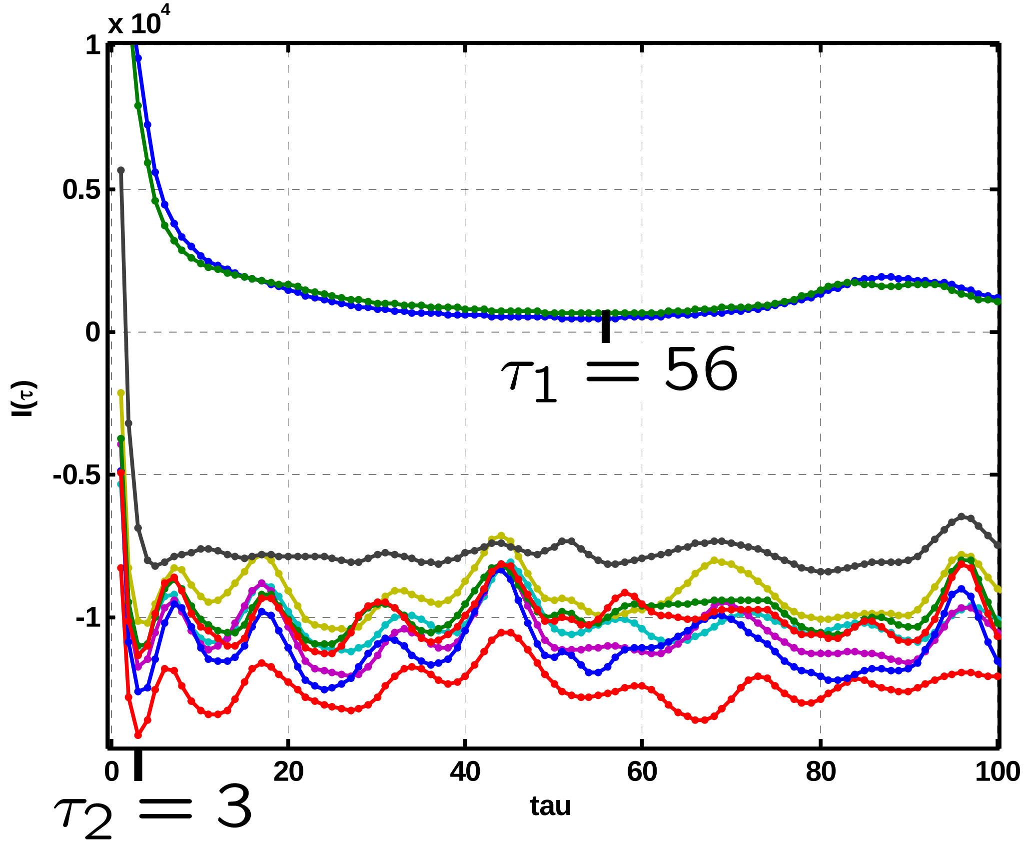
→ embedding suffices to render the (local) structure of the system dynamics

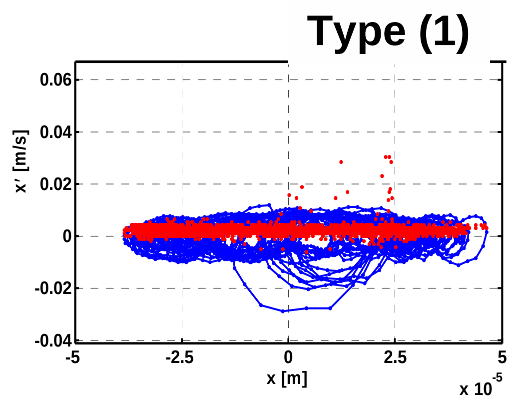


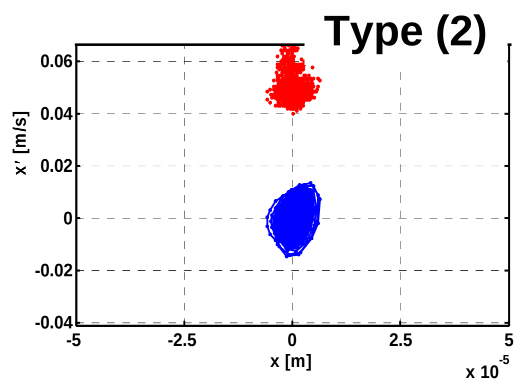
Embedding Dimension n :
minimal phase-space dimension

Correlation Dimension $D_C(n,r)$:
(local) attractor dimension

Time Delay τ **:** first minimum of the Average Mutual Information







(Discussion of other common criteria: please refer to paper)