

## Kolloquium für Mechanik

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Titel: **Enforcing physical bounds in finite volume and discontinuous Galerkin phase-field simulations**

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### Abstract

The Cahn–Hilliard equation is (besides the Allen–Cahn equation) the prototype phase-field model for immiscible mixtures of two fluids. This fourth-order nonlinear parabolic PDE describes phase separation at a constant temperature in the presence of a mass constraint and the dissipation of free energy, eventually building two ‘pure’ bulk phases, which are separated by a smooth interface. The order parameter,  $u$ , which is the main unknown, is physically meaningful only if it attains values within a certain interval, e.g., in  $[-1,1]$  when  $u$  is chosen as the difference of mass fractions of either fluid component. Whether analytical solutions  $u$  satisfy the corresponding bound preservation inequalities,  $-1 \leq u(t,x) \leq 1$ , depends on the chosen potential function  $\Psi(u)$  appearing in the equation. Such potentials that guarantee bound preservation do not allow input values of  $u$  outside the admissible range and thus lead to the termination of a simulation if overshoots or undershoots occur in the numerical solution. To enforce the global bounds of  $-1$  and  $1$  for finite volume solutions or for element averages of discontinuous Galerkin (DG) solutions, the numerical fluxes must be constrained in an appropriate manner. To this end, we propose a novel flux limiter that has to be applied after each time step and that is computed by local (time-explicit) iterations. In the case of DG, after the mean values are bound-preserved, the higher-order solution parts can be controlled by a subsequent slope limiter.

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Alle Interessenten sind herzlich eingeladen.  
Prof. Dr.-Ing. Bettina Frohnappel