Nonuniform Transformation Field Analysis of composites with high morphological anisotropy

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INTRODUCTION

The effective properties of physically non-linear composite materials are investigated using the *nonuniform transformation field analysis (NTFA)* introduced by **Michel and Suquet (2003, 2004)**. An example for the class of examined materials are metal ceramic composites, where ceramic particles are used as a reinforcement embedded into an elasto-plastic matrix material. The influence of *anisotropic particle morphologies* on the physically non-linear effective material response is investigated.

NONUNIFORM TRANSFORMATION FIELD ANALYSIS (**Michel and Suquet (2003,2004); Fritzen and Böhlke (2010a)**)

Finite-dimensional approximation of the *inelastic strain*

$$\boldsymbol{\varepsilon}^{\mathrm{p}}(t,\boldsymbol{x}) \approx \sum_{\alpha=1}^{N} \xi_\alpha(t)\boldsymbol{\mu}^{(\alpha)}(\boldsymbol{x})$$

$\boldsymbol{\mu}^{(\alpha)}$: inelastic modes, $\hat{\xi} \in \mathbb{R}^N$: mode activity coefficients
Induced stress and strain fields for given macro-strain $\langle \boldsymbol{\varepsilon} \rangle = \bar{\boldsymbol{\varepsilon}}$

$$\boldsymbol{\varepsilon}(t,\boldsymbol{x}) = \mathbb{A}(\boldsymbol{x})[\bar{\boldsymbol{\varepsilon}}(t)] + \sum_{\alpha=1}^{N} \xi_\alpha(t)\boldsymbol{\varepsilon}_*^{(\alpha)}(\boldsymbol{x})$$

$$\boldsymbol{\sigma}(t,\boldsymbol{x}) = \mathbb{C}(\boldsymbol{x})\mathbb{A}(\boldsymbol{x})[\bar{\boldsymbol{\varepsilon}}(t)] + \sum_{\alpha=1}^{N} \xi_\alpha(t)\boldsymbol{\sigma}_*^{(\alpha)}(\boldsymbol{x})$$

Thermodynamic driving forces ($\boldsymbol{B}^{(\alpha)}$: orthonormal basis of Sym)

$$(\hat{A})_{\alpha\beta} = \langle \mathbb{A}^{\mathsf{T}}\boldsymbol{\sigma}_*^{(\alpha)} \rangle \cdot \boldsymbol{B}^{(\beta)}, \quad (\hat{D})_{\alpha\beta} = \langle \boldsymbol{\mu}^{(\alpha)} \cdot \boldsymbol{\sigma}_*^{(\beta)} \rangle, \quad \hat{\tau} = \hat{A}\hat{\bar{\varepsilon}} + \hat{D}\hat{\xi}$$

Rate of internal variables

$$\dot{\hat{\xi}} = \dot{\lambda}\frac{\partial \bar{\varphi}}{\partial \hat{\tau}}, \qquad \dot{\lambda} \geq 0, \qquad \bar{\varphi} \leq 0, \qquad \bar{\varphi}(\hat{\tau}) = \|\hat{\tau}\|_2 - k\sigma_{\mathrm{F}}(\bar{q}) \leq 0$$

Effective material response for $\bar{\varepsilon} \equiv \bar{\varepsilon}(t), \hat{\xi} \equiv \hat{\xi}(t)$

$$\bar{\mathbb{C}} = \langle \mathbb{C}\mathbb{A} \rangle, \qquad \bar{\boldsymbol{\sigma}}_*^{(\alpha)} = \langle \boldsymbol{\sigma}_*^{(\alpha)} \rangle, \qquad \bar{\boldsymbol{\sigma}}(\bar{\boldsymbol{\varepsilon}}, \hat{\xi}) = \bar{\mathbb{C}}[\bar{\boldsymbol{\varepsilon}}] + \sum_{\alpha=1}^{N} \xi_\alpha \bar{\boldsymbol{\sigma}}_*^{(\alpha)}$$

Algorithmic tangent operator

$$\bar{\mathbb{C}}_{\mathrm{a}} = \bar{\mathbb{C}} + \sum_{\alpha=1}^{N} \langle \boldsymbol{\sigma}_*^{(\alpha)} \rangle \otimes \frac{\partial \xi_\alpha}{\partial \bar{\boldsymbol{\varepsilon}}}$$

ANISOTROPIC MICROSTRUCTURES

(cf. **Fritzen and Böhlke (2011a,b)**)

AIM: Isolate the influence of the particle morphology on the **effective non-linear response**

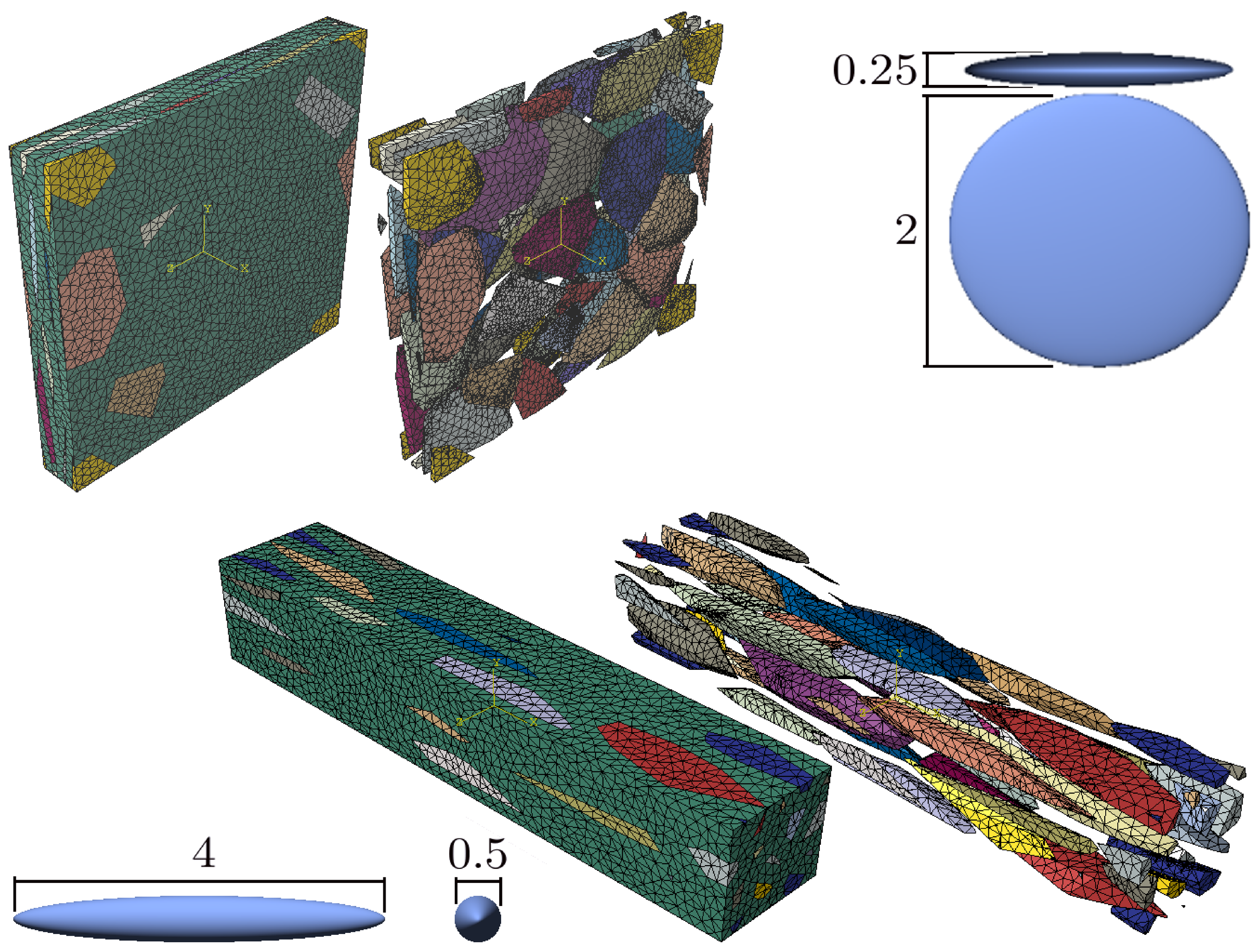

Fig. 1: Anisotropic particle reinforced materials

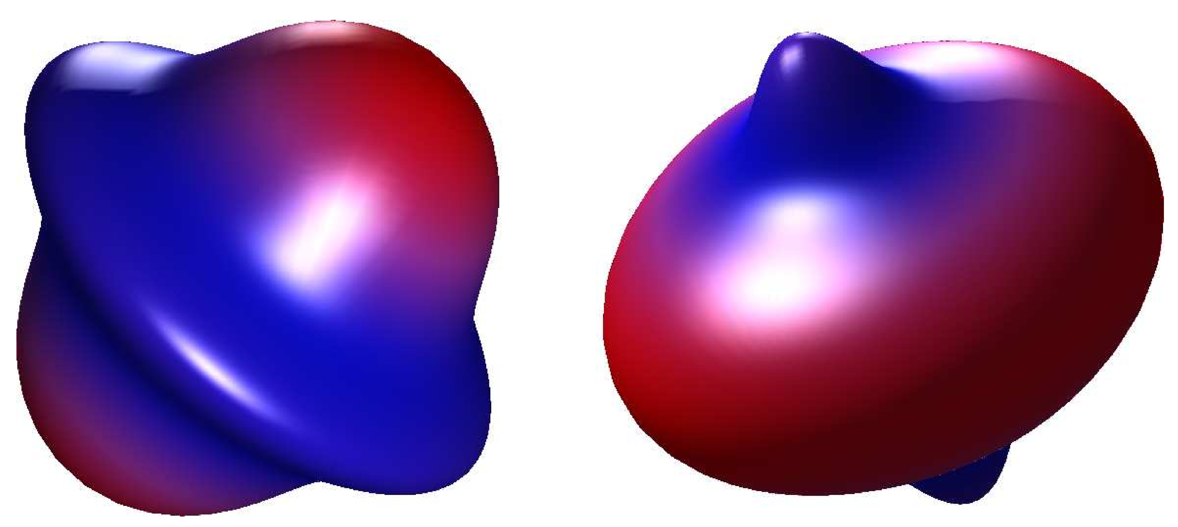

Fig. 2: Effective yield surfaces of the NTFA model

RESULTS (cf. **Fritzen and Böhlke (2011b)**)

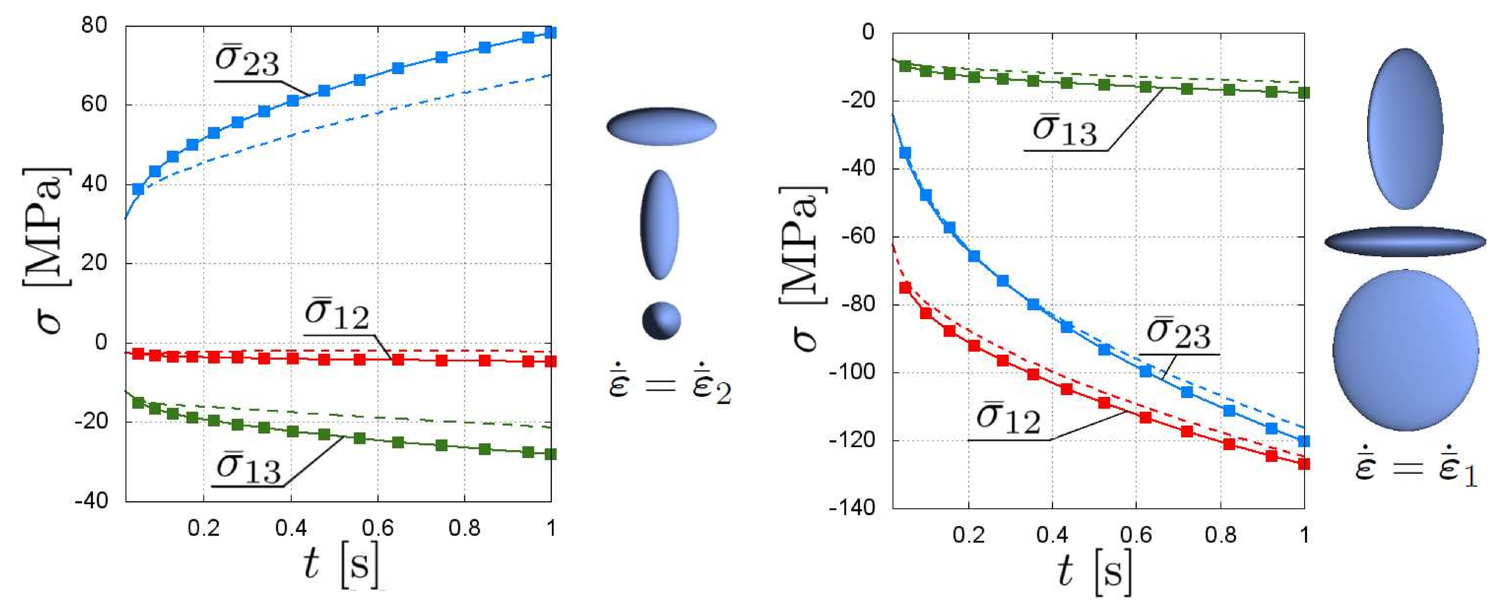

Fig. 3: Comparison of $\bar{\sigma}$: NTFA vs. full-field simulations

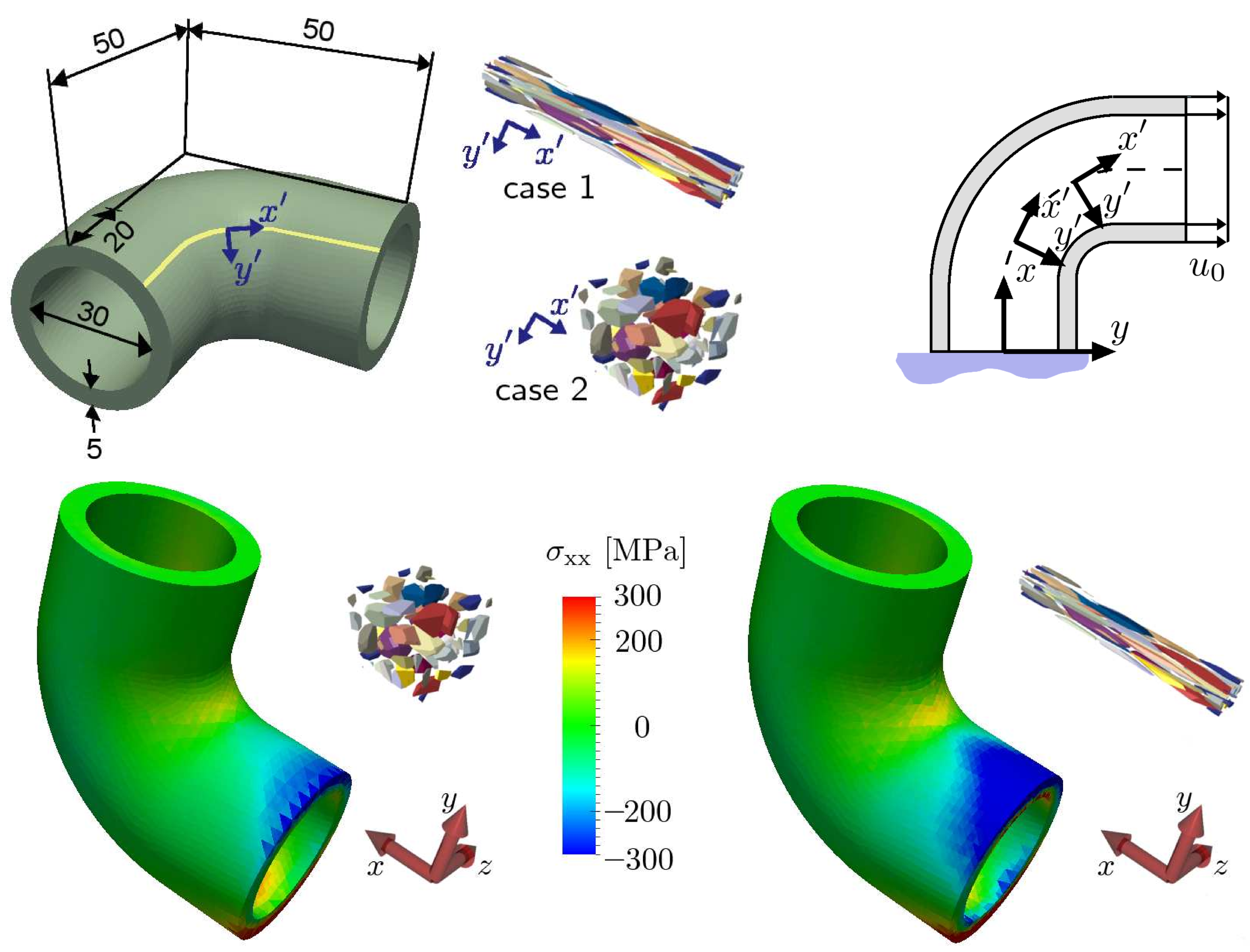

Fig. 4: Influence of the morphology on the macroscopic stress (*left:* isotropic particles, *right:* elongated particles)

REFERENCES

Michel, J.-C. and Suquet, P. (2003) International Journal of Solids and Structures **40**, 2003
Michel, J.-C. and Suquet, P. (2004) Computer Methods in Applied Mechanics and Engineering **193**, 2004
Fritzen, F. and Böhlke, T. (2010a) International Journal for Numerical Methods in Engineering **84**, 2010
Fritzen, F. and Böhlke, T. (2011a) International Journal of Solids and Structures **48**, 2011
Fritzen, F. and Böhlke, T. (2011b) Composite Science and Technology **71**, 2011