7th ESMC, Lisboa, 2009

On self-excited vibrations due to sliding friction in systems of deformable bodies

Towards a general formulation & influence of contact properties

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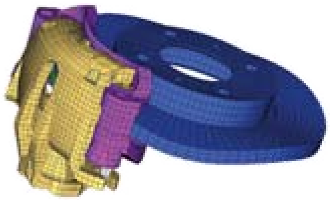
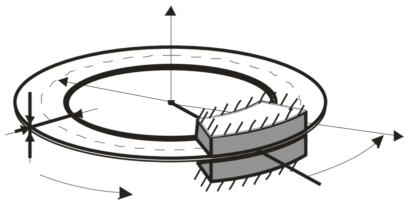
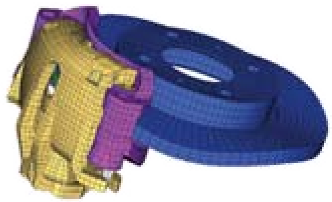


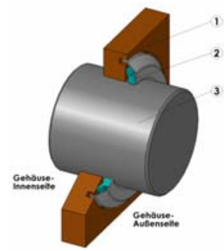


Motivation

Photos: www.technik-handel.com, Bosch

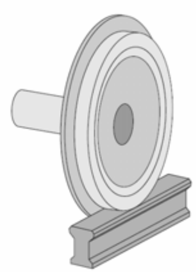


vehicle brakes



shaft seals

self-excited vibrations



train wheels

grinding tools

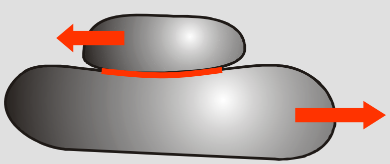
- **steady-state stability**
- **linear perturbation equations**

usually:
- FE-models
- structural models
- → perturbation equations
- → specific formulations

?

- general results
- further effects
- parameters

abstract system



generic perturbation equations

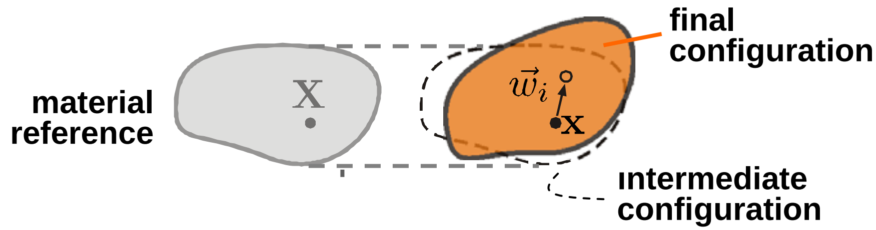
for
- systems of continua
- sliding friction contacts
- spatially fixed contact zone

System description

Hamilton's Principle
(for open systems)

$$\boxed{\int_0^t \left\{ \delta L + \delta W_{np}^* - \underbrace{\delta \Pi_C}_{\text{normal contact}} + \underbrace{\delta W_{\Gamma_C}}_{\text{tangential contact, sliding friction}} \right\} \mathrm{d}t = 0} \qquad L = T - U$$

Spatial description



- Eulerian coordinates of intermediate configuration $\mathbf{x} = \mathbf{x}\left(\mathbf{X}, t\right)$
- rigid body motion relates $\mathbf{x} = \mathbf{X} + \mathbf{v}_T t$
- small vibrations $\vec{w}_i = \vec{r}_i - \vec{r}_{Ti}$ about rigid body motion → **Linearization**

$$\boxed{0 = \sum_{i=1}^{N} \int_{\Omega_i} \delta\vec{w}_i \cdot \left(\mathcal{M}_i[\ddot{\vec{w}}_i] + \mathcal{P}_i[\dot{\vec{w}}_i] + \mathcal{Q}_i[\vec{w}_i] \right) \mathrm{d}v + \underbrace{\Delta\left\{\delta\Pi_C\right\} - \Delta\left\{\delta W_{\Gamma_C}\right\}}_{\textbf{linearized contact contributions}} - \Delta\left\{\delta W_{np}\right\}}$$

$$\mathcal{P}_i = \mathcal{D}_i + v_{Tij}\mathcal{G}_{ij}$$
$$\mathcal{Q}_i = \mathcal{K}_i + v_{Tij}\mathcal{N}_{ij} + v_{Tij}v_{Tik}\mathcal{K}_{ijk}^*$$

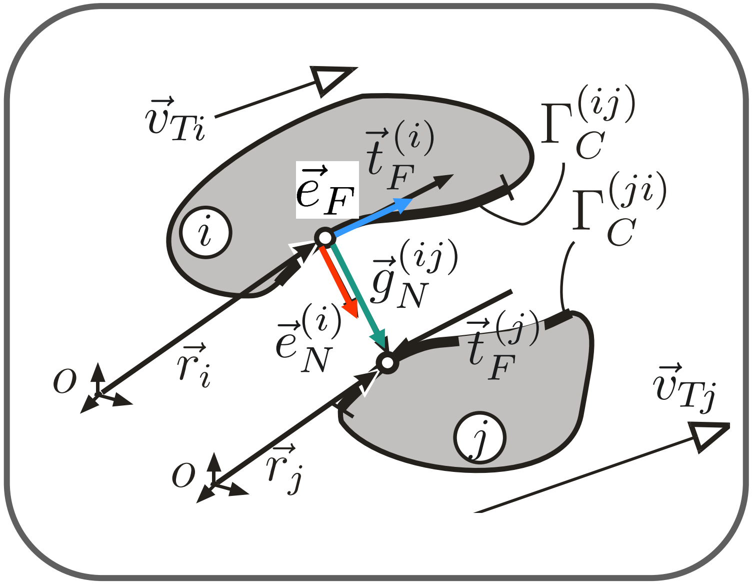
Contact

Kinematics

surface normal $\vec{e}_N^{(i)}$

gap vector

$$\vec{g}^{(ij)} = \vec{r}_j - \vec{r}_i = (\vec{r}_{Tj} - \vec{r}_{Ti}) + (\vec{w}_j - \vec{w}_i)$$
$$= \vec{g}_0^{(ij)} + \Delta\vec{g}^{(ij)}$$



normal distance

$$g_N^{(ij)} = \vec{e}_N^{(i)} \cdot \vec{g}^{(ij)} = g_{N0}^{(ij)} + \Delta g_N^{(ij)}$$

$$\Delta g_N^{(ij)} = \vec{e}_N^{(i)} \cdot \Delta\vec{g}^{(ij)}$$
$$\delta g_N^{(ij)} = \vec{e}_N^{(i)} \cdot \delta\vec{g}^{(ij)}$$

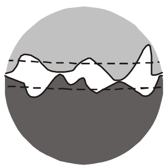
friction direction $\vec{e}_F = \vec{v}_{rel} / |\vec{v}_{rel}|$

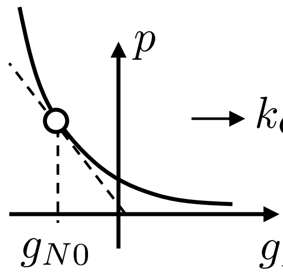
Contact

$$0=\sum_{i=1}^{N}\int_{\Omega_i}\delta\vec{w}_i\cdot\Big(\mathcal{M}_i[\ddot{\vec{w}}_i]+\mathcal{P}_i[\dot{\vec{w}}_i]+\mathcal{Q}_i[\vec{w}_i]\Big)\,\mathrm{d}v+\boxed{\Delta\{\delta\Pi_C\}-\Delta\{\delta W_{\Gamma_C}\}}+\Delta\{\delta W_{np}\}$$

$$\Delta\{\}=\{\}-\{\}\big|_0$$

Normal contact



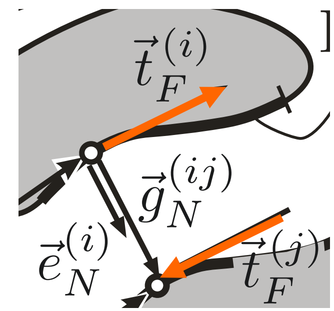


$$\rightarrow k_C=\frac{\partial p}{\partial g_N}\Big|_0$$

$$\Delta\{\delta\Pi_C\}\quad=\quad\int_{\Gamma_C^{(ij)}}\delta\vec{g}\,\big[k_C\vec{e}_N\otimes\vec{e}_N\big]_0\Delta\vec{g}\,\mathrm{d}a$$

Friction

$$\delta W_C=\int_{\Gamma_C^{(ij)}}\delta\vec{g}\cdot\vec{e}_F\,\mu p_N\,\mathrm{d}a$$



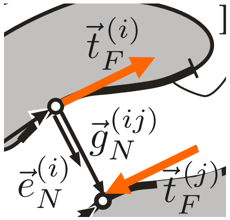
$$-\Delta\{\delta W_C\}=\int_{\Gamma_C^{(ij)}}\delta\vec{g}\cdot\Big(\big[\mu p_N\big]_0\Delta\vec{e}_F+\big[\mu\vec{e}_F\big]_0\Delta p_N\Big)\,\mathrm{d}a$$

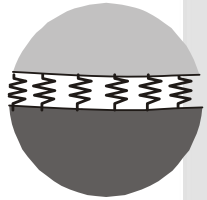
$$=\int_{\Gamma_C^{(ij)}}\delta\vec{g}\cdot\Bigg(\underbrace{\left[\frac{\mu p_N}{v_{rel}}\right]_0\Delta\dot{\vec{g}}+\left[\frac{\mu p_N}{v_{rel}}\right]_0\left(\mathbf{v}_{Tj}^{\top}\frac{\partial\vec{w}_j}{\partial\mathbf{x}}-\mathbf{v}_{Ti}^{\top}\frac{\partial\vec{w}_i}{\partial\mathbf{x}}\right)}_{\text{orientation of friction vector}}-\underbrace{\big[\mu k_C(\vec{e}_F\otimes\vec{e}_N)\big]_0\Delta\vec{g}}_{\text{contact pressure}}\Bigg)\,\mathrm{d}a$$

Universität Karlsruhe (TH)
Research University · founded 1825

Contact contributions

$$0 = \sum_{i=1}^{N} \int_{\Omega_i} \delta\vec{w}_i \cdot \Big(\mathcal{M}_i[\ddot{\vec{w}}_i] + \mathcal{P}_i[\dot{\vec{w}}_i] + \mathcal{Q}_i[\vec{w}_i] \Big)\, \mathrm{d}v + \boxed{\Delta\{\delta\Pi_C\} - \Delta\{\delta W_{\Gamma_C}\}} - \Delta\{\delta W_{np}\}$$





$$\Delta\{\delta\Pi_C\} - \Delta\{\delta W_{\Gamma_C}\}$$

$$= \int_{\Gamma_C^{(ij)}} \delta\vec{g} \cdot \Big(\left[k_C(\vec{e}_N \otimes \vec{e}_N)\right]_0 \Delta\vec{g}$$

← **normal contact** **frictional contact**

$$+ \left[\frac{\mu p_N}{v_{rel}}\right]_0 \Delta\dot{\vec{g}} + \left[\frac{\mu p_N}{v_{rel}}\right]_0 \left(\mathbf{v}_{Tj}^{\top} \frac{\partial \vec{w}_j}{\partial \mathbf{x}} - \mathbf{v}_{Ti}^{\top} \frac{\partial \vec{w}_i}{\partial \mathbf{x}}\right) - \left[\mu k_C(\vec{e}_F \otimes \vec{e}_N)\right]_0 \Delta\vec{g} \Big)\, \mathrm{d}a$$

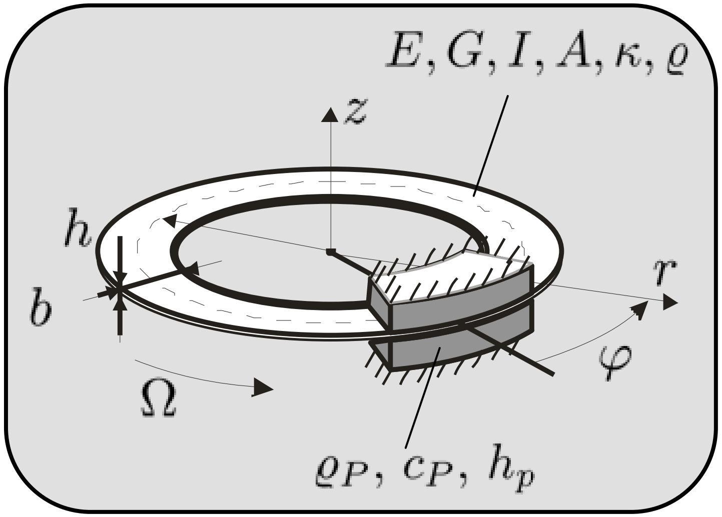
→ **normal contact**
- stiffness
- symmetric, pos. semidefinite

→ **friction**
- damping and „stiffness“
- damping: symmetric, positive semidefinite, grows with 1/v
- stiffness: non-symmetric

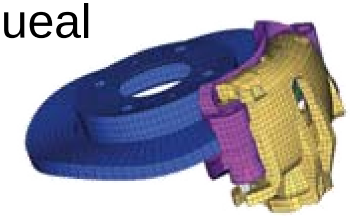
parameters $\mu,\ k_C,\ p_N,\ v_{rel},\ \Gamma_C$

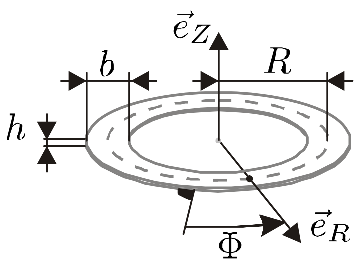
- **no discretization**
- **no structural model**

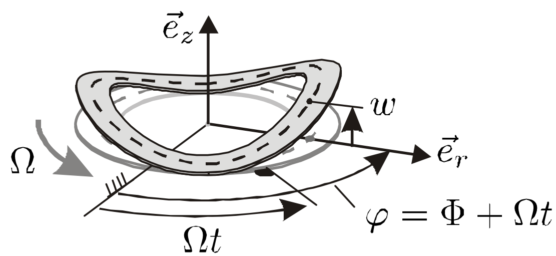
Example: rotating Timoshenko ring



- rotating circular Timoshenko beam
- friction pads as Winkler foundation
- Eulerian description
- simple model for brake squeal







$$\varphi = \Phi + \Omega t$$

data: $\kappa = 5/6,\ R = 0.12\text{m},\ h = 0.01\text{m},\ b = 0.1\text{m},\ \varphi_0 = \pi/8,$
$c_P = 8 \cdot 10^8 \text{Pa/m},\ h_p = 0.02\text{m},\ k_p = 5 \cdot 10^{10}\ \text{Pa/m},$
$E = 2.1 \cdot 10^{11}\text{Pa},\ \nu = 0.33,\ \varrho = 7800\text{kg/m}^3$

Stability of steady-state

$$0 = \sum_{i=1}^{N} \int_{\Omega_i} \delta\vec{w}_i \cdot \left(\mathcal{M}_i[\ddot{\vec{w}}_i] + \Omega\mathcal{G}_i[\dot{\vec{w}}_i] + \mathcal{D}_{S,i}[\dot{\vec{w}}_i] + \mathcal{K}_{S,i}[\vec{w}_i] \right) \mathrm{d}v + \boxed{\Delta\{\delta\Pi_C\} - \Delta\{\delta W_{\Gamma_C}\}} - \Delta\{\delta W_{np}\}$$

$$= \sum_{(ij)}^{n_C} \int_{\Gamma_C^{(ij)}} \delta\vec{g} \cdot \left(k_C\,(\vec{e}_N \otimes \vec{e}_N)\,\Delta\vec{g} + \frac{\mu p_0}{v_T}\Delta\dot{\vec{g}} + \mu k_C\,(\vec{e}_F \otimes \vec{e}_N)\,\Delta\vec{g} \right) \mathrm{d}a$$

Ritz-discretization

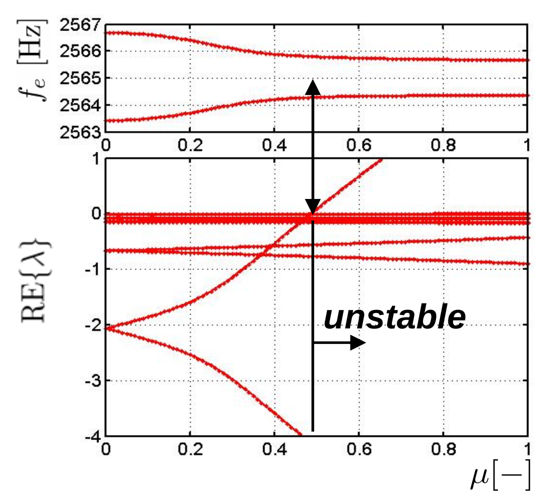
$\vec{w}_i \approx \mathbf{\Phi}\mathbf{q}_i$

$$\boxed{\mathbf{M}\ddot{\mathbf{q}} + \left(\Omega\mathbf{G} + \mathbf{D}_S + \frac{\mu p_0}{\Omega}\mathbf{R}_1\right)\dot{\mathbf{q}} + \left(\mathbf{K}_S + k_C\mathbf{K}_N + \mu k_C h\mathbf{R}_3\right)\mathbf{q} = \mathbf{0}}$$

$$\mathbf{R}_1 = \mathbf{R}_1^\top \qquad \mathbf{K}_N = \mathbf{K}_N^\top \qquad \mathbf{R}_3 = \mathbf{R}_3^\top$$

- **circulatory system**
  - flutter instability
  - extremely sensible to damping and gyroscopic terms
- **depends on**

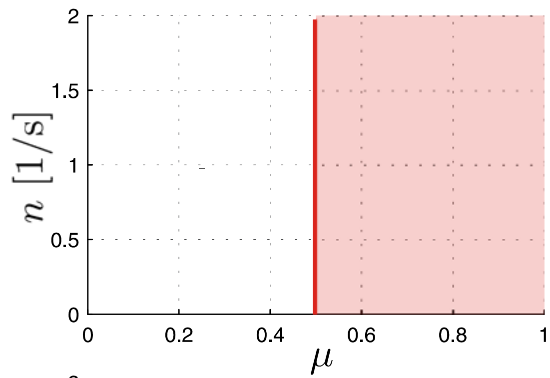
$$\mu,\ k_C,\ p_N,\ \Omega,\ \Gamma_C$$



Stability: damping & gyroscopic terms

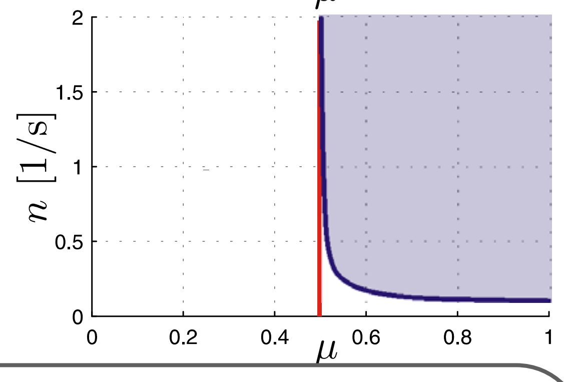
„Friction damping“ and gyroscopic terms

$$\mathbf{M}\ddot{\mathbf{q}} + \left(\quad + \mathbf{D}_S \quad \right)\dot{\mathbf{q}} + \left(\mathbf{K} + \mu k_C h \mathbf{R}_3\right)\mathbf{q} = \mathbf{0}$$



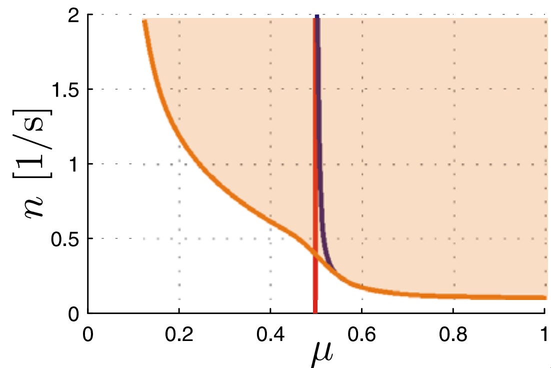
$$\mathbf{M}\ddot{\mathbf{q}} + \left(\quad + \mathbf{D}_S + \frac{\mu p_0}{\Omega}\mathbf{R}_1\right)\dot{\mathbf{q}} + \left(\mathbf{K} + \mu k_C h \mathbf{R}_3\right)\mathbf{q} = \mathbf{0}$$

- friction contribution to damping important at low relative speeds



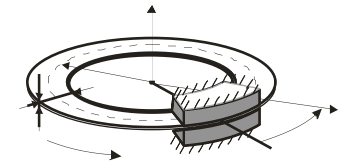
$$\mathbf{M}\ddot{\mathbf{q}} + \left(\Omega\mathbf{G} + \mathbf{D}_S + \frac{\mu p_0}{\Omega}\mathbf{R}_1\right)\dot{\mathbf{q}} + \left(\mathbf{K} + \mu k_C h \mathbf{R}_3\right)\mathbf{q} = \mathbf{0}$$

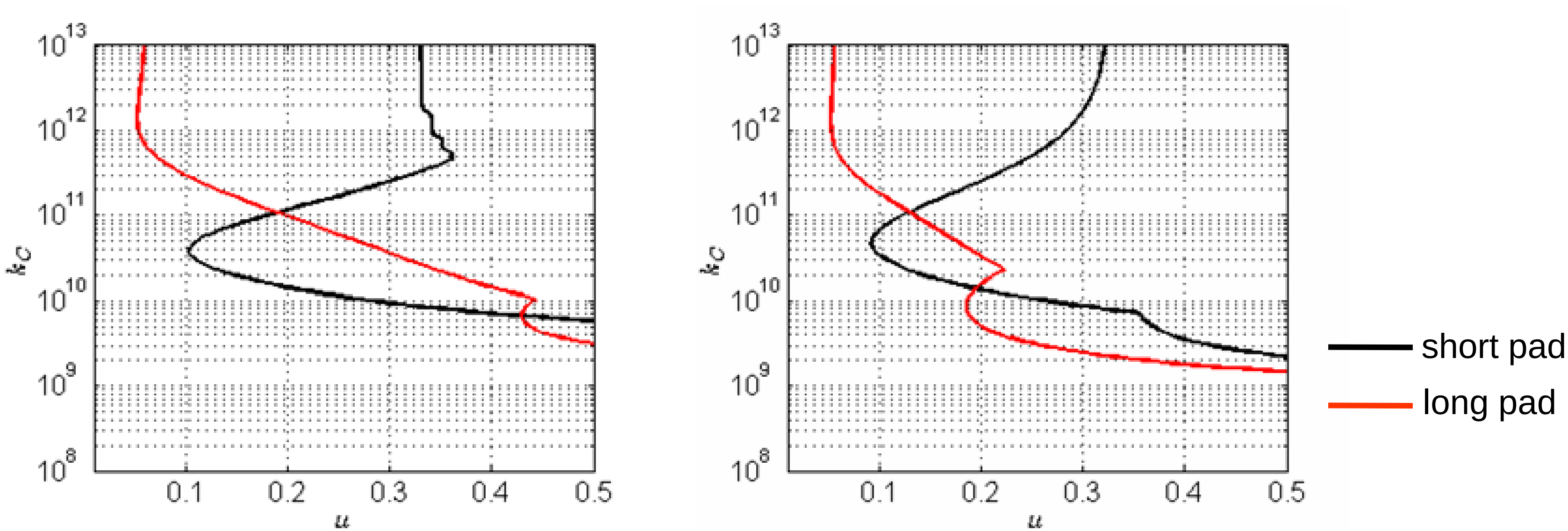
- gyroscopic terms have significant effect
- Theorem of Thomson&Tait does not apply to circulatory systems!



Contact stiffness

$$\mathbf{M}\ddot{\mathbf{q}} + \left(\Omega\mathbf{G} + \mathbf{D}_S + \frac{\mu p_0}{\Omega}\mathbf{R}_1\right)\dot{\mathbf{q}} + \left(\mathbf{K} + \boxed{\mu k_C h}\mathbf{R}_3\right)\mathbf{q} = \mathbf{0}$$

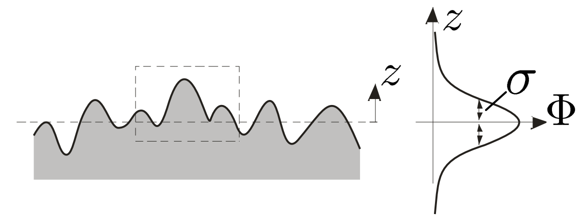




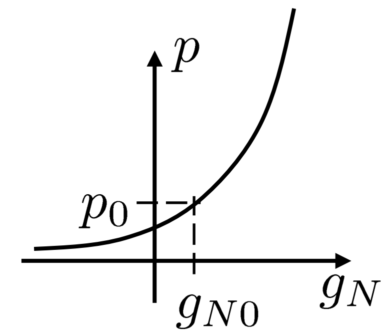
- strong influence of contact stiffness
- more than a sheer „penalty parameter“

Tribology / Constitutive contact law

Greenwood-Williamson



- rough surface
- here: Gaussian distribution for asperity heights, std. deviation σ
- local deformation: Hertz-theory

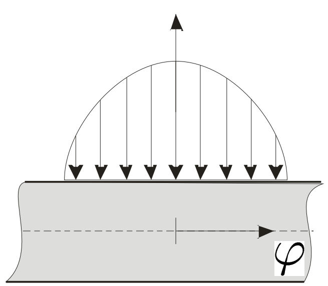


$$\longrightarrow \boxed{p(g_N) = p_0 \mathrm{e}^{\frac{3}{2\sigma}\gamma(g_N - g_{N0})}} \quad \gamma = \gamma(h_0) \approx 1..2$$

$$p(g_{N0} + \Delta g_N) = p_0 + \underbrace{\left[p_0 \frac{3}{2\sigma}\gamma\right]}\Delta g_N + \mathcal{O}(2)$$

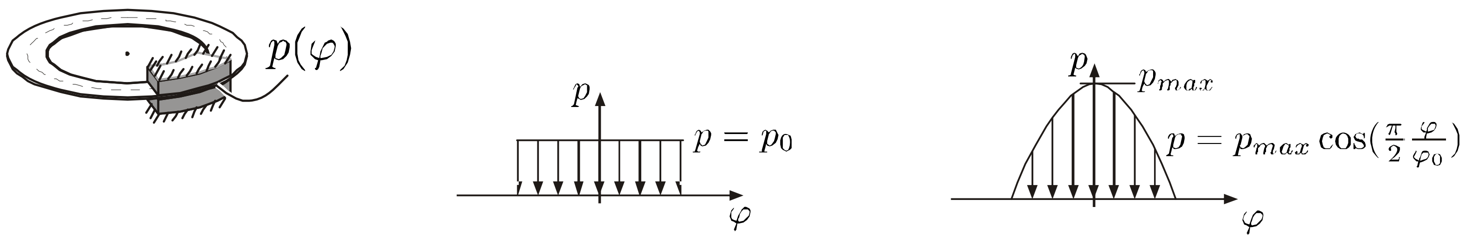
p_0 - steady-state contact pressure

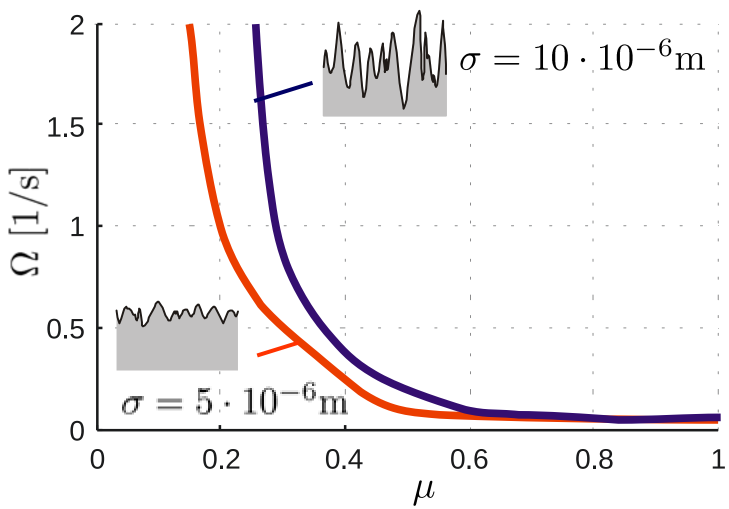
- pressure distribution $p_0 = p_0(\varphi)$
- usually $\gamma \approx 4/3 ... 6/3$

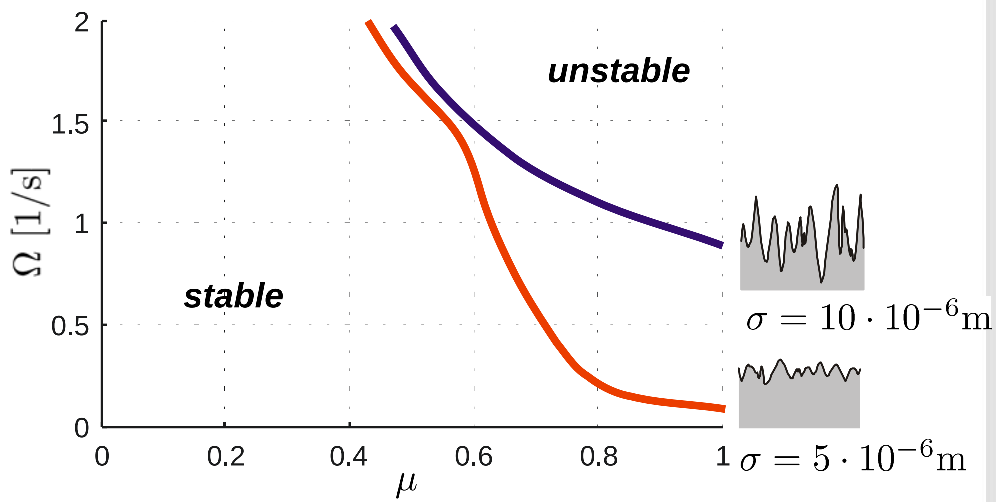


$$\boxed{k_C(\varphi, \sigma) \approx p_0(\varphi)\frac{5}{2\sigma}}$$

Stability: Contact pressure & topography

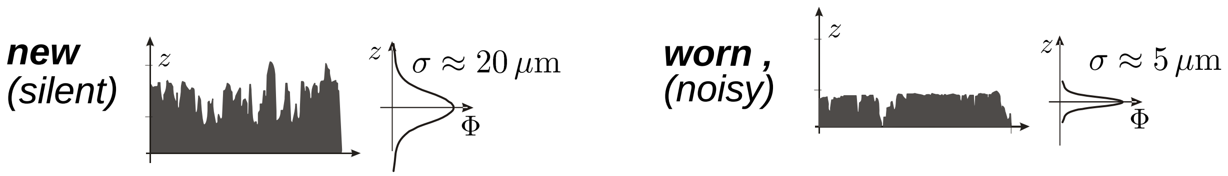






Experimental data

(disk brake squeal)

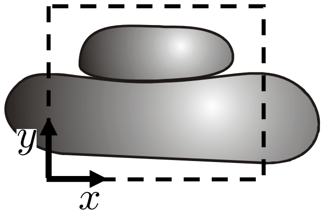


(Sherif: *Investigation on effect of surface topography…on squeal generation*, Wear, 2004)

Conclusion

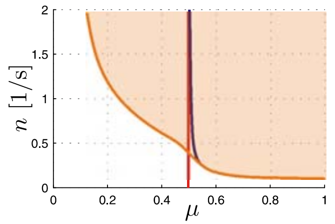
Abstract problem

- generic form of perturbation equations
- physical meaning of the contributions
- no discretization / no structural model
- systematic way to formulate contributions



Example: moving Timoshenko-Ring

- rotating timoshenko-ring
- strong influence of transport motion and „friction damping“
- contact stiffness shows strong influence
  → microcmechanics of contact need to be considered



Thank you for your attention!